re-Algebra I (Chapter 6)
Whole Numbers
Whole Number Computation

5, 6, 7, 8

Divide.

13. 1006.83 ÷ 1.13
14. 486 ÷ 0.391
15. 0.909 ÷ 1.35
16. 5.005 ÷ 0.095

Compute.

17. (56.4 × 7) + (8 ÷ 2)
18. (34.9 − 24.5) × 2.6
19. (19.4 + 5.4 − 8.3) × 2
20. 53.4 ÷ 17.8 − 3

(Answers on page 335)

ACT-TYPE PROBLEMS

1. One day the high temperature is 92.6°; the low is 77.8°. What is the difference between the high and low temperatures on this day?

   A. 16.6°
   B. 14.8°
   C. 14.4°
   D. 13.8°
   E. 13.6°

2. On a mathematics test, Jordan scored 87.7 and Corey scored 79.4. Find the average of Jordan's and Corey's test scores.

   F. 85
   G. 84.5
   H. 83.55
   J. 83
   K. 82.25

3. Alex is a carpenter and he has fifty 1.5-inch nails and fifty 0.75-inch nails. What would be the length of all these nails if they were lined up end to end?

   A. 112.5 inches
   B. 115 inches
   C. 117.5 inches
   D. 119 inches
   E. 120 inches

4. Gene weighed 170 pounds. Then he worked out for 5 days, followed a new diet, and lost 0.8 pound a day. How much did Gene weigh after the 5 days?

   F. 163 pounds
   G. 164 pounds
   H. 165 pounds
   J. 166 pounds
   K. 167 pounds

5. The odometer on Eileen's car reads 2,004.7 miles. What will the odometer read at the end of the day on Friday after she drives the distances shown below?

Day	Miles
Monday	5.6 miles
Tuesday	10.4 miles
Wednesday	7.8 miles
Thursday	11.2 miles
Friday	22.7 miles

   A. 2,006.2 miles
   B. 2,062.4 miles
   C. 2,074.7 miles
   D. 2,045.6 miles
   E. 2,004.5 miles

(Answers on page 335)

Factors, Divisibility, and Primes

Factors

A **factor** of any number divides the number exactly, with no remainder. Since 45 ÷ 9 = 5 and 45 ÷ 5 = 9, 5 and 9 are factors of 45. Thinking of it another way, 5 × 9 = 45, so 5 and 9 are factors of 45.

Every number has 1 and itself as factors.

Factors of 1 are: 1

Factors of 2 are: 1 and 2

Factors of 4 are: 1, 2, and 4

Factors of 15 are: 1, 3, 5, and 15

When discussing factors and primes, we just consider the whole numbers beginning with 1.

Divisibility Rules

Divisibility rules can be used to determine whether a number is divisible (can be divided exactly so there is no remainder) by another number. You can also use your calculator.

Use these rules to determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

1 Every number is divisible by 1.

2 Even numbers, which end in 0, 2, 4, 6, or 8, are divisible by 2.

3 If the sum of the digits is divisible by 3, then the number is divisible by 3.

Examples

2,481	1,033
sum of the digits:	sum of the digits:
2 + 4 + 8 + 1 = 15	1 + 0 + 3 + 3 = 7
15 is divisible by 3, so	7 is not divisible by 3, so
2,481 is divisible by 3.	1,033 is not divisible by 3.

4 If the last two digits are divisible by 4, then the number is divisible by 4.

Examples

26,347,464	45,344,694
64 is divisible by 4, so	94 is not divisible by 4, so 45,344,694
26,347,464 is divisible by 4.	is not divisible by 4.

5 If the last digit is 0 or 5, then the number is divisible by 5.

6 If a number is divisible by 2 and 3, then the number is divisible by 6.

7 Divisibility rule is more complex than just dividing by 7.

8 If the last three digits are divisible by 8, then the number is divisible by 8.

Examples

91,384,656	56,400,686
656 is divisible by 8, so	686 is not divisible by 8, so
91,384,656 is divisible by 8.	56,400,686 is not divisible by 8.

9 If the sum of the digits is divisible by 9, then the number is divisible by 9.

Examples

86,715	93,163
sum of the digits:	sum of the digits:
8 + 6 + 7 + 1 + 5 = 27	9 + 3 + 1 + 6 + 3 = 22
27 is divisible by 9, so	22 is not divisible by 9, so
86,715 is divisible by 9.	93,163 is not divisible by 9.

10 If a number ends in 0, then the number is divisible by 10.

Divisibility rules help you simplify fractions and equations.

Prime and Composite Numbers

Factors of a number exactly divide that number.

A **prime number** has exactly two factors, 1 and itself.

A **composite number** has more than two factors.

Look at some numbers, starting with 1, to see if they are prime or composite.

1 has only one factor, itself. 1 is neither prime nor composite.
2 has exactly two factors, 1 and 2. 2 is a prime number, the only even prime.
3 has exactly two factors, 1 and 3. 3 is a prime number.
4 has more than two factors: 1, 2, and 4. 4 is a composite number.
5 has exactly two factors, 1 and 5. 5 is a prime number.
6 has more than two factors: 1, 2, 3, and 6. 6 is a composite number.
7 has exactly two factors, 1 and 7. 7 is a prime number.
8 has more than two factors: 1, 2, 4, and 8. 8 is a composite number.
9 has more than two factors: 1, 3, and 9. 9 is a composite number.

The prime numbers less than 30 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

MODEL ACT PROBLEM

Which of the following numbers is not a prime number?

A. 37
B. 43
C. 51
D. 67
E. 89

SOLUTION

A prime number has only itself and 1 as factors. Use divisibility rules to find that 51 is divisible by 3—3 is a factor of 51. Since 51 has more than two factors, it is not a prime number.

The correct answer is C.

Practice

Determine whether 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are factors of each number.

1. 105 **2.** 111 **3.** 96 **4.** 176

List all the factors of each number.

5. 54 **6.** 67 **7.** 87 **8.** 120

Is the number prime or composite? Explain your answer.

9. 1 **10.** 205 **11.** 41 **12.** 111,111,111
13. 127 **14.** 123,123 **15.** 119 **16.** 675,201

(Answers on page 335)

ACT-TYPE PROBLEMS

1. What is the sum of all the factors of 20?

A. 12
B. 21
C. 31
D. 33
E. 42

2. Which of the following is the product of the numbers from 1 to 9 that evenly divide 473,124?

F. 144
G. 192
H. 1,152
J. 8,064
K. 10,368

3. All of the following choices are composite numbers EXCEPT:

A. 17
B. 18
C. 81
D. 116
E. 117

4. What is the sum of the prime numbers between 30 and 40?

F. 0
G. 64
H. 68
J. 76
K. 107

5. Which of the following choices is a fac 235,783,784?

A. 3
B. 4
C. 5
D. 6
E. 7

(Answers on page 336)

Square Roots, Exponents, and Scientific Notation

Square Roots

The square root of a number multiplied by itself equals the number.

This symbol means the square root of 64: $\sqrt{64}$

The square root of 64 is 8 because $8 \times 8 = 64$.

A square root may be a whole number. The numbers with whole number square roots are called **perfect squares**.

$\sqrt{1} = 1$ $\sqrt{4} = 2$ $\sqrt{9} = 3$ $\sqrt{16} = 4$ $\sqrt{25} = 5$ $\sqrt{36} = 6$
$\sqrt{49} = 7$ $\sqrt{64} = 8$ $\sqrt{81} = 9$ $\sqrt{100} = 10$ $\sqrt{121} = 11$ $\sqrt{144} = 12$

You can also find the square root of a decimal.

$\sqrt{1.44} = 1.2$ because $1.2 \times 1.2 = 1.44$ $\sqrt{1.69} = 1.3$ because $1.3 \times 1.3 = 1.69$

CALCULATOR TIP

Use the $\sqrt{\ }$ key on your calculator to find square roots. If the number is not a perfect square, the $\sqrt{\ }$ key will give you an approximation for the square root.
For example, entering $\sqrt{\ }$ 15 will give you 3.8729833 ...
3.87 is an approximation of the square root of 15.

Exponents

An exponent shows repeated factors.

exponent
↓
base → 3^4

The **base** shows the factor. The **exponent** shows how many times the factor is repeated.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

The fractional exponent $x^{\frac{1}{2}}$ is another way of writing square root.

$81^{\frac{1}{2}} = \sqrt{81} = 9$

$3.61^{\frac{1}{2}} = \sqrt{3.61} = 1.9$

Any non-zero number raised to the zero power is equal to 1.

$15^0 = 1$

$237^0 = 1$

Negative exponents show fractions and decimals.

$x^{-n} = \frac{1}{x^n}\ (x \neq 0)$

$11^{-1} = \frac{1}{11^1} = \frac{1}{11}$

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1{,}000} = 0.001$

CALCULATOR TIP

The exponent key on a calculator is usually labeled [y^x] or [^].

To find 4^3, enter 4 [y^x] 3 [=] or 4 [^] 3 [=].

Scientific Notation

To write a positive number in scientific notation, multiply a decimal between 1 and 10 by a power of 10.

EXAMPLES

$6{,}728 = 6.728 \times 10^3$

Move the decimal point **three** places to the **left** and use 10^3.

$0.056 = 5.6 \times 10^{-2}$

Move the decimal point **two** places to the **right** and use 10^{-2}.

MODEL ACT PROBLEM

Light travels about 186,756 miles in a second. Which of the following shows about how far light travels in 100 seconds?

A. 1.8675600×10^{-8} miles
B. 1.86756×10^{-7} miles
C. 1.86756×10^{7} miles
D. 1.86756×10^{8} miles
E. 1.8675600×10^{9} miles

SOLUTION

Calculate how far light travels in 100 seconds.

$186{,}756 \times 100 = 18{,}675{,}600$ miles

Write 18,675,600 in scientific notation.

Move the decimal point seven places left and use 10^7.

1.86756×10^7

The correct answer is C.

Practice

Write the value.

1. $\sqrt{400}$ **2.** $\sqrt{0.25}$ **3.** $\sqrt{324}$ **4.** $\sqrt{2.89}$

5. 10^3 **6.** 4^3 **7.** 12^0 **8.** $49^{\frac{1}{2}}$

9. 2^{-1} **10.** 6^{-2} **11.** 10^{-3} **12.** $16^{\frac{1}{2}}$

Write each number in scientific notation.

13. 2,325,000,000 **14.** 0.0175 **15.** 2.75 **16.** 5,659

17. 0.062 **18.** 50,790 **19.** 0.00257 **20.** 6,894,590

Write each as a whole number or decimal.

21. 4.2×10^5 **22.** 5.01×10^3 **23.** 2.35×10^{-2} **24.** 6.6×10^2

25. 9.09×10^{-3} **26.** 6.7×10^4 **27.** 4.05×10^{-4} **28.** 1.9×10^4

(Answers on page 336)

ACT-TYPE PROBLEMS

1. The area of a square is 5.76 square inches. What is the length of each side of the square?

   A. 1.44
   B. 2.3
   C. 2.4
   D. 2.5
   E. 2.73

2. Which of the following numbers, multiplied by itself 4 times, equals 2,401?

   F. 6
   G. 7
   H. 8
   J. 9
   K. 10

3. The width of a virus is 0.0000001 meter. Which of the following correctly represents that measurement?

   A. 10^{-1} meters
   B. 10^{-5} meters
   C. 10^{-6} meters
   D. 10^{-7} meters
   E. 10^{-8} meters

4. Which of the following is the closest approximation of $15^{\frac{1}{2}}$?

   F. 3.6
   G. 3.7
   H. 3.8
   J. 3.9
   K. 4.0

5. It is about six billion kilometers from Earth to Pluto. Which of the following choices correctly represents that distance?

   A. 10^{9} kilometers
   B. 6×10^{9} kilometers
   C. 10^{10} kilometers
   D. 6×10^{10} kilometers
   E. 6^{10} kilometers

(Answers on page 336)

■ Fractions

Fractions are numerals that can name part of a whole. The **denominator** of a fraction shows how many equal parts or objects in all. The **numerator** shows how many parts are being discussed.

$$\frac{\text{numerator}}{\text{denominator}} \begin{matrix}\rightarrow 7 \\ \rightarrow 8\end{matrix}$$

The fraction $\frac{7}{8}$ can mean 7 out of 8 objects, 7 of 8 equal parts of a whole, or seven-eighths of the way from 0 to 1 on a number line. A stock price of $\frac{7}{8}$ means seven-eighths of a dollar.

> A fraction can't have a denominator of 0. If a fraction turns up with a denominator of 0, we say it is undefined. For example, $\frac{7}{0}$ is undefined.

Equivalent Fractions

The fraction $\frac{7}{8}$ is a name for a number. Other fractions that name the same number are called **equivalent fractions**. To find an equivalent fraction, multiply or divide the numerator and denominator by the same number.

EXAMPLES

$\frac{7 \times 5}{8 \times 5} = \frac{35}{40}$ $\quad$ $\frac{7}{8}$ is equivalent to $\frac{35}{40}$.

$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$ $\quad$ $\frac{8}{12}$ is equivalent to $\frac{4}{6}$.

Simplest Form

A fraction is in **simplest form** when the numerator and denominator have no common factors greater than 1.

$\frac{3}{8}$ is in simplest form. No number greater than 1 divides both 3 and 8 exactly.

$\frac{14}{35}$ is not in simplest form. The numerator and the denominator are both divisible by 7.

$\frac{14 \div 7}{35 \div 7} = \frac{2}{5}$. $\frac{2}{5}$ is in simplest form.

CALCULATOR TIP

Use a calculator that can represent fractions and mixed numbers. The calculator can also convert fractions to simplest form and convert improper fractions to mixed numbers.

Comparing Fractions

Use the terms equivalent to ($a = b$), less than ($a < b$), greater than ($a > b$), and between ($a < b < c$) to compare fractions. If two fractions have the same denominator, the fraction with the greater numerator is greater.

$\frac{4}{8} = \frac{4}{8}$ $\quad$ $\frac{1}{8} < \frac{3}{8}$ $\quad$ $\frac{5}{8} > \frac{4}{8}$ $\quad$ $\frac{3}{8} < \frac{6}{8} < \frac{7}{8}$

EXAMPLE

Compare $\frac{3}{8}$ and $\frac{5}{12}$.

You can always cross multiply to compare fractions.
The greater cross product appears next to the greater fraction.

$$^{36}\frac{3}{8} \times \frac{5}{12}{}^{40}$$

$$\frac{3}{8} < \frac{5}{12}$$

> You can also convert to a common denominator to compare fractions.

15, 16

CALCULATOR TIP

Convert fractions to decimals for easy comparison. A calculator that can represent fractions will convert quickly between fractions and decimals. If you do not have this kind of calculator, divide the numerator of the fraction by the denominator to change it to a decimal.

MODEL ACT PROBLEM

In lowest terms, what is the difference between the largest and smallest fraction listed below?

$\frac{1}{2}, \frac{3}{5}, \frac{4}{7}, \frac{2}{3}$

A. $\frac{2}{21}$
B. $\frac{1}{8}$
C. $\frac{1}{6}$
D. $\frac{9}{14}$
E. $\frac{41}{42}$

SOLUTION

Use a calculator for this problem.

Enter:

1 [÷] 2 [=]	0.5
3 [÷] 5 [=]	0.6
4 [÷] 7 [=]	0.5714285714
2 [÷] 3 [=]	0.6666666667

The largest fraction is $\frac{2}{3} = 0.\overline{6}$ and the smallest fraction is $\frac{1}{2} = 0.5$. Subtract the smallest from the largest. Entering 2 [÷] 3 [−] 1 [÷] 2 [=] gives $0.1\overline{6}$ or $\frac{1}{6}$. The answer is C.

Practice

Write True if the fractions are equivalent and False if they are not equivalent.

1. $\frac{4}{7}$ is equivalent to $\frac{12}{21}$
2. $\frac{3}{5}$ is equivalent to $\frac{6}{15}$
3. $\frac{5}{9}$ is equivalent to $\frac{20}{36}$
4. $\frac{2}{3}$ is equivalent to $\frac{12}{18}$
5. $\frac{1}{5}$ is equivalent to $\frac{4}{25}$
6. $\frac{4}{9}$ is equivalent to $\frac{20}{45}$
7. $\frac{3}{4}$ is equivalent to $\frac{12}{18}$
8. $\frac{5}{7}$ is equivalent to $\frac{35}{49}$

Write each fraction in simplest form.

9. $\frac{27}{81}$
10. $\frac{8}{12}$

Fill in the blank with <, >, or =.

11. $\frac{3}{5}$ ___ $\frac{4}{7}$
12. $\frac{2}{9}$ ___ $\frac{5}{12}$
13. $\frac{2}{3}$ ___ $\frac{1}{2}$ ___ $\frac{3}{8}$
14. $\frac{3}{4}$ ___ $\frac{6}{7}$ ___ $\frac{7}{8}$

Write in order from least to greatest.

15. $\frac{7}{8}, \frac{5}{6}$
16. $\frac{3}{5}, \frac{61}{100}$
17. $\frac{2}{7}, \frac{4}{9}$
18. $\frac{3}{5}, \frac{11}{20}, \frac{4}{7}$
19. $\frac{3}{4}, \frac{5}{8}, \frac{7}{10}$
20. $\frac{4}{5}, \frac{1}{2}, \frac{7}{16}, \frac{5}{9}$

(Answers on page 336)

ACT-TYPE PROBLEMS

1. All of the following fractions are equivalent to $\frac{4}{7}$ EXCEPT:

A. $\frac{8}{14}$
B. $\frac{12}{35}$
C. $\frac{16}{28}$
D. $\frac{36}{63}$
E. $\frac{24}{42}$

2. Which of the following choices correctly lists the fractions $\{\frac{3}{5}, \frac{5}{6}, \frac{3}{10}, \frac{1}{3}, \frac{8}{15}\}$ from least to greatest?

F. $\frac{5}{6}, \frac{3}{10}, \frac{1}{3}, \frac{8}{15}, \frac{3}{5}$
G. $\frac{5}{6}, \frac{3}{5}, \frac{3}{10}, \frac{8}{15}, \frac{1}{3}$
H. $\frac{3}{10}, \frac{1}{3}, \frac{8}{15}, \frac{3}{5}, \frac{5}{6}$
J. $\frac{3}{5}, \frac{5}{6}, \frac{3}{10}, \frac{1}{3}, \frac{8}{15}$
K. $\frac{3}{5}, \frac{3}{10}, \frac{8}{15}, \frac{1}{3}, \frac{5}{6}$

3. For which of the following choices would the symbol = in the blank make the statement true?

A. $\frac{2}{5}$ ___ $\frac{4}{15}$
B. $\frac{7}{13}$ ___ $\frac{21}{39}$
C. $\frac{5}{9}$ ___ $\frac{20}{27}$
D. $\frac{3}{7}$ ___ $\frac{12}{21}$
E. $\frac{5}{12}$ ___ $\frac{50}{1,200}$

4. Joel has 5 different kinds of salad to choose from. He has \$2 to spend. Which of the 5 salads listed below should Joel choose to get the most salad for his money?

Salad	Amount for \$2
Mediterranean salad	$\frac{1}{2}$ pound
Caesar salad	$\frac{2}{3}$ pound
Garden salad	$\frac{5}{7}$ pound
Oriental salad	$\frac{4}{9}$ pound
Potato salad	$\frac{3}{8}$ pound

F. Mediterranean salad
G. Caesar salad
H. Garden salad
J. Oriental salad
K. Potato salad

5. Five college students, Brad, Alex, Scott, Jim, and Tony, live in the same dorm. The following is a list of the distances each walks to class. Which student walks the farthest?

Name	Distance
Brad	$\frac{7}{15}$ of a mile
Alex	$\frac{8}{13}$ of a mile
Scott	$\frac{9}{17}$ of a mile
Jim	$\frac{4}{9}$ of a mile
Tony	$\frac{5}{12}$ of a mile

A. Brad
B. Alex
C. Scott
D. Jim
E. Tony

(Answers on page 337)

Pre-Algebra I (Chapter 6)

Addition and Subtraction of Fractions and Mixed Numbers

Follow these steps to add or subtract fractions and mixed numbers.

- Estimate first.
- Compute.
- Remember to write the answer in simplest form.
- Check your answer against the estimate to be sure your answer is reasonable.

Sometimes an estimate may be enough to answer the question.

A **mixed number** has a whole number part and a fraction part.

$2\frac{3}{4}$ is a mixed number.

CALCULATOR TIP

On a calculator that can represent mixed numbers, estimate first, then complete the keystrokes to show the answer in simplest form.

If you do not have this kind of calculator, express the mixed fraction as the sum of the integer part and the fractional part. For example, $7\frac{1}{5} = 7 + \frac{1}{5}$.

Addition

EXAMPLES

1. $\frac{2}{3} + \frac{3}{5}$

Estimate first.

Both fractions are closer to 1 than to 0.

The answer should be between 1 and 2.

Use common denominators.	Add the numerators.	Write in simplest form. Is the answer reasonable?
$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$	$\frac{10}{15}$	
$\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$	$+\frac{9}{15}$	
	$\frac{19}{15}$	$\frac{19}{15} = 1\frac{4}{15}$
		$1\frac{4}{15}$ is between 1 and 2.
		The answer is reasonable.

2. $1\frac{3}{4} + 7\frac{1}{3}$

Estimate first.

One fraction is less than a half and the other is more than a half.

The sum of the whole numbers is 8.

The answer should be close to 9.

Use common denominators.	Add the fractions and the whole numbers.	Write in simplest form. Is the answer reasonable?
$1\frac{3}{4} = 1\frac{9}{12}$	$1\frac{9}{12}$	
$7\frac{1}{3} = 7\frac{4}{12}$	$+\ 7\frac{4}{12}$	
	$8\frac{13}{12}$	$8\frac{13}{12} = 8 + 1\frac{1}{12} = 9\frac{1}{12}$
		The answer is reasonable. It is close to 9.

Subtraction

EXAMPLES

1. $14\frac{2}{3} - 3\frac{1}{7}$

Estimate first.

The fractional part of the difference is a little less than $\frac{2}{3}$.

The difference between the whole numbers is 11.

The answer should be between 11 and $11\frac{2}{3}$.

Use common denominators.	Subtract the numerators and the whole numbers.	Write in simplest form. Is the answer reasonable?
$14\frac{2}{3} = 14\frac{14}{21}$	$14\frac{14}{21}$	
$3\frac{1}{7} = 3\frac{3}{21}$	$-\ 3\frac{3}{21}$	
	$11\frac{11}{21}$	The answer is in simplest form. The answer is reasonable. It is between 11 and $11\frac{2}{3}$.

2. $5\frac{1}{5} - 2\frac{5}{6}$

Estimate first.

$\frac{5}{6}$ is larger than $\frac{1}{5}$ so you will have to borrow from 5 to subtract.

The difference between the fractions will be less than 1.

Borrowing 1 from 5, you are left with 4. The difference between 4 and 2 is 2.

The answer should be between 2 and 3.

Write fractions with common denominators, then rename again.	Subtract.	Write in simplest form. Is the answer reasonable?
$5\frac{1}{5} = 5\frac{6}{30} = 4\frac{36}{30}$	$4\frac{36}{30}$	
$2\frac{5}{6} = 2\frac{25}{30} = 2\frac{25}{30}$	$-2\frac{25}{30}$	
	$2\frac{11}{30}$	The answer is in simplest form. The answer is reasonable. It is between 2 and 3.

MODEL ACT PROBLEM

The following table shows how far Frank rode his bike this week.

Day	*Mileage*
Monday	$2\frac{3}{4}$ miles
Wednesday	$\frac{7}{8}$ mile
Thursday	$\frac{3}{5}$ mile
Friday	$3\frac{1}{2}$ miles

What is the total number of miles that Frank rode on Monday, Wednesday, and Friday?

A. $6\frac{3}{8}$

B. $6\frac{1}{2}$

C. $7\frac{1}{8}$

D. $7\frac{1}{2}$

E. 8

SOLUTION

Use only the distances for Monday $\left(2\frac{3}{4}\right)$, Wednesday $\left(\frac{7}{8}\right)$, and Friday $\left(3\frac{1}{2}\right)$.

Use your calculator to add the fractions. Or write the fractions with common denominators and add.

$2\frac{3}{4} = 2\frac{6}{8}$, $\quad \frac{7}{8}$, $\quad 3\frac{1}{2} = 3\frac{4}{8}$

Add the fraction parts and then add the whole number parts.

$$\begin{array}{r} 2\frac{6}{8} \\ \frac{7}{8} \\ +3\frac{4}{8} \\ \hline 5\frac{17}{8} \end{array}$$

$5\frac{17}{8} = 5 + 2\frac{1}{8} = 7\frac{1}{8}$

The correct answer is C.

Practice

Estimate first. Then compute.

Add.

1. $\frac{3}{7} + \frac{2}{3}$
2. $\frac{4}{9} + \frac{5}{18}$
3. $\frac{6}{13} + \frac{1}{3}$
4. $\frac{11}{28} + \frac{6}{7}$
5. $\frac{5}{8} + \frac{2}{3}$
6. $\frac{1}{2} + \frac{1}{6} + \frac{5}{8}$
7. $\frac{3}{4} + \frac{1}{5} + \frac{7}{10}$
8. $\frac{2}{15} + \frac{2}{3} + \frac{1}{5}$
9. $8\frac{1}{3} + 6\frac{3}{7}$
10. $6\frac{2}{3} + 9\frac{5}{7}$
11. $14\frac{7}{8} + 6\frac{3}{4}$
12. $27\frac{5}{6} + 14\frac{3}{4}$

Subtract.

13. $\frac{9}{17} - \frac{1}{2}$
14. $\frac{12}{19} - \frac{7}{18}$
15. $\frac{4}{7} - \frac{3}{10}$
16. $\frac{8}{15} - \frac{2}{5}$
17. $\frac{2}{3} - \frac{1}{10}$
18. $4\frac{7}{8} - 2\frac{5}{6}$
19. $25 - 8\frac{4}{5}$
20. $6\frac{5}{8} - 2\frac{2}{3}$
21. $11\frac{3}{5} - 6\frac{3}{4}$
22. $10\frac{4}{5} - 3\frac{1}{3}$
23. $46 - 31\frac{3}{4}$
24. $21\frac{3}{8} - 16\frac{5}{6}$

(Answers on page 337)

ACT-TYPE PROBLEMS

1. Fast and Thrifty Shipping Company charges $6 a pound for packages. A company has a shipment of three packages weighing $\frac{3}{5}$ pound, $1\frac{4}{15}$ pounds, and $\frac{23}{30}$ pound. How much will it cost to ship these packages by Fast and Thrifty?

   A. $15.00
   B. $15.80
   C. $16.00
   D. $16.20
   E. $16.60

2. Jill bought some fruit at the grocery store. She bought $2\frac{1}{2}$ pounds of apples, $\frac{3}{4}$ pound of bananas, $1\frac{2}{5}$ pounds of peaches, and $5\frac{7}{10}$ pounds of watermelon. What is the total number of pounds of fruit that Jill bought at the store?

   F. 9
   G. $9\frac{1}{2}$
   H. $10\frac{5}{8}$
   J. $10\frac{7}{20}$
   K. $11\frac{13}{30}$

3. Alice needs a rope at least 8 feet long. She has four smaller ropes of the following lengths:

   I. $4\frac{2}{3}$ feet
   II. $4\frac{2}{5}$ feet
   III. $3\frac{3}{4}$ feet
   IV. $3\frac{2}{3}$ feet

   Alice can tie the ropes together. However, when she does so she loses a total of $\frac{1}{4}$ of a foot. Which of the following combinations of rope will NOT give Alice a piece of rope at least 8 feet long?

   A. I and II
   B. I and III
   C. II and III
   D. I and IV
   E. I, II, and III

TOPIC Griffin Ray

lgebra I (Chapter 6)

4. All of the following equal 1 EXCEPT:

F. $\frac{1}{2} + \frac{3}{12} + \frac{9}{36}$

G. $\frac{2}{3} + \frac{2}{6}$

H. $\frac{3}{8} + \frac{20}{24} - \frac{1}{6}$

J. $\frac{13}{7} - \frac{12}{14}$

K. $\frac{3}{9} + \frac{1}{3} + \frac{4}{12}$

5. Rich, Brian, Jack, and Chad ordered a pizza that is cut into eight slices of the same size. Rich ate two slices, Brian ate one slice, and Jack ate three slices. What fraction of the pie was left for Chad to eat?

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

E. $\frac{3}{4}$

(Answers on page 337)

Multiplication and Division of Fractions and Mixed Numbers

Follow these steps to multiply or divide fractions and mixed numbers.

- Estimate first.
- Compute.
- Remember to write the answer in simplest form.
- Check your answer against the estimate to be sure your answer is reasonable.

Sometimes an estimate may be enough to answer the question.

> You use multiplication to solve both multiplication and division problems.

Multiplication

EXAMPLES

1. $\frac{2}{3} \times \frac{3}{8}$

Estimate first.

Both fractions are less than 1.

The answer will be less than either of the fractions.

Multiply numerators. Multiply denominators.	Write the answer in simplest form.	Check to be sure your answer is reasonable.
$\frac{2}{3} \times \frac{3}{8} = \frac{6}{24}$	$\frac{6}{24} = \frac{1}{4}$	$\frac{1}{4}$ is less than and $\frac{2}{3}$ and $\frac{3}{8}$, so the answer is reasonable.

2. $1\frac{1}{5} \times 2\frac{7}{8}$

Estimate first.

You are multiplying a little more than 1 by almost 3.

The answer will be between 3 and 4.

Write mixed numbers as fractions.	Multiply numerators. Multiply denominators.	Write in simplest form. Is the answer reasonable?
$1\frac{1}{5} = \frac{6}{5}$ $2\frac{7}{8} = \frac{23}{8}$	$\frac{6}{5} \times \frac{23}{8} = \frac{138}{40}$	$\frac{138}{40} = 3\frac{18}{40} = 3\frac{9}{20}$ $3\frac{9}{20}$ is between 3 and 4. The answer is reasonable.

Division

To divide fractions, invert the divisor and multiply.

EXAMPLES

1. $\frac{3}{4} \div \frac{1}{3}$

Estimate first.

Think: Invert $\frac{1}{3}$ to get 3. You are multiplying a little less than 1 by 3. The answer should be between 2 and 3.

Invert the divisor.	Multiply.	Write in simplest form. Is the answer reasonable?
$\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1}$	$\frac{3}{4} \times \frac{3}{1} = \frac{9}{4}$	$\frac{9}{4} = 2\frac{1}{4}$ $2\frac{1}{4}$ is between 2 and 3, so the answer is reasonable.

> Watch the first step. Many mistakes may occur when you invert the divisor.

2. $6\frac{1}{2} \div 1\frac{3}{4}$

Estimate first.

$6\frac{1}{2}$ is near 6 and $1\frac{3}{4}$ is near 2.

$6 \div 2 = 3$. The answer should be near 3.

Write mixed numbers as fractions.	Invert the divisor.	Multiply.	Write in simplest form. Is the answer reasonable?
$6\frac{1}{2} = \frac{13}{2}$ $1\frac{3}{4} = \frac{7}{4}$	$\frac{13}{2} \div \frac{7}{4} = \frac{13}{2} \times \frac{4}{7}$	$\frac{13}{2} \times \frac{4}{7} = \frac{52}{14}$	$\frac{52}{14} = 3\frac{10}{14} = 3\frac{5}{7}$ $3\frac{5}{7}$ is close to 3, so the answer is reasonable.

MODEL ACT PROBLEM

Which of the following statements is *false* about (a) $\frac{1}{2}$ and (b) $\frac{1}{2}$?

A. The product of (a) and (b) is less than (a).
B. The sum of (a) and (b) is greater than the product of (a) and (b).
C. (a) divided by (b) is equal to the sum of (a) and (b).
D. (a) minus (b) is less than 0.
E. The product of (a) and (b) is less than (a) divided by (b).

SOLUTION

Work out each choice.

A.	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	True
B.	$\frac{1}{2} + \frac{1}{2} = 1$ and 1 is greater than $\frac{1}{4}$.	True
C.	$\frac{1}{2} \div \frac{1}{2} = 1$ and $\frac{1}{2} + \frac{1}{2} = 1$. 1 equals 1.	True
D.	$\frac{1}{2} - \frac{1}{2} = 0$ and 0 is NOT less than 0.	False
E.	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and $\frac{1}{2} \div \frac{1}{2} = 1$. $\frac{1}{4} < 1$.	True

The correct answer is D.

Practice

Multiply.

1. $\frac{2}{3} \times \frac{1}{5}$
2. $\frac{7}{8} \times \frac{4}{9}$
3. $15 \times \frac{4}{5}$
4. $3\frac{3}{4} \times \frac{5}{8}$
5. $5\frac{1}{8} \times \frac{24}{25} \times 10\frac{1}{2}$
6. $4\frac{2}{7} \times 5\frac{1}{6}$
7. $3\frac{1}{8} \times 4\frac{1}{5}$
8. $2\frac{1}{4} \times 6\frac{1}{2} \times \frac{12}{39}$

Divide.

9. $\frac{2}{3} \div \frac{3}{5}$
10. $\frac{3}{4} \div \frac{3}{8}$
11. $\frac{5}{6} \div 10$
12. $\frac{8}{\frac{1}{8}}$
13. $\frac{\frac{2}{3}}{\frac{3}{8}}$
14. $5\frac{1}{3} \div 2\frac{2}{3}$
15. $7\frac{3}{5} \div 6\frac{1}{2}$
16. $5\frac{1}{3} \div 4\frac{1}{6}$
17. $\frac{2}{3} \times \frac{1}{2} \div 3$
18. $\frac{4}{5} \div \frac{3}{10} \times \frac{1}{2}$
19. $\frac{11}{15} \times 10 \div 2 \times \frac{1}{3}$
20. $\frac{4}{7} \times \frac{28}{3} \div 4$

(Answers on page 338)

ACT-TYPE PROBLEMS

1. Bret runs $\frac{2}{3}$ of a mile. Tony runs half as far as Bret. How many miles does Tony run?
   - **A.** $1\frac{1}{2}$
   - **B.** $1\frac{1}{3}$
   - **C.** $1\frac{1}{6}$
   - **D.** $\frac{1}{3}$
   - **E.** $\frac{1}{6}$

2. Jim bench-presses 225 pounds and does arm curls with a weight $\frac{1}{3}$ the amount he bench-presses. What weight does he arm-curl?
   - **F.** 65 pounds
   - **G.** 70 pounds
   - **H.** 75 pounds
   - **J.** 80 pounds
   - **K.** 85 pounds

3. $\frac{2}{5} + \left(\frac{25}{9} \div \frac{1}{3}\right) = ?$
   - **A.** $\frac{3}{10}$
   - **B.** $\frac{13}{25}$
   - **C.** $3\frac{1}{3}$
   - **D.** $8\frac{11}{15}$
   - **E.** $10\frac{1}{5}$

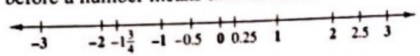
4. A college math professor has to grade 160 exams. He gives $\frac{1}{2}$ of the exams to his assistant to grade. Then the assistant gives $\frac{1}{4}$ of her exams to a student worker to grade. How many exams does the student worker get to grade?
   - **F.** 20 exams
   - **G.** 40 exams
   - **H.** 60 exams
   - **J.** 70 exams
   - **K.** 80 exams

5. $\frac{5}{7} \div \frac{15}{3} \times \frac{11}{13} = ?$
   - **A.** $\frac{12}{51}$
   - **B.** $\frac{11}{91}$
   - **C.** $\frac{5}{71}$
   - **D.** $\frac{23}{57}$
   - **E.** $\frac{17}{73}$

(Answers on page 338)

Positive and Negative Numbers

You can visualize positive and negative numbers on a number line. A plus sign (+) before a number means that the number is to the right of zero. A negative sign (−) before a number means that the number is to the left of zero.



Write positive fifteen. 15

Write negative eleven. −11

> You do not need to show the sign if the number is positive.

Absolute Value

The absolute value of a number is positive and is the distance from zero to the number. The symbol for the absolute value of a is $|a|$.

EXAMPLES

$|5| = 5$ $\quad |-5| = 5$ $\quad |0| = 0$ $\quad |19 - 37| = |-18| = 18$

Griffin Key

5, 6, 7, 8
9, 10

Comparing Positive and Negative Numbers

Integers, decimals, and fractions can be positive or negative. Use these rules to compare positive and negative numbers.

- If the signs of the numbers are different, the positive number is greater.

  $\frac{1}{16} > -28$ $\quad -53 < 7.8$ $\quad 10.3 > -807$

- If both signs are positive, compare the numbers.

  $203.3 > 203.198$ $\quad \frac{1}{2} < \frac{3}{4}$ $\quad 71 > 65\frac{1}{2}$

- If both signs are negative, the larger numeral represents the smaller number.

  $-908 < -8$ $\quad -\frac{1}{7} > -\frac{7}{8}$ $\quad -108.53 < 6.01$

MODEL ACT PROBLEM

Which of the following statements is false?

A. $-123.45 > -90.7$
B. $3 < 6.7$
C. $-34 < 2$
D. $17 > -600$
E. $17.89 < 17.9$

SOLUTION

In choice A, both signs are negative. Therefore, the larger numeral represents the smaller number.
$-123.45 < -90.7$

The correct answer is A.

Practice

Find the absolute value.

1. $|-8|$ **2.** $|58 - 41|$ **3.** $|13 + 12|$ **4.** $|17|$

5. $|41 - 7|$ **6.** $|-54|$ **7.** $|13.5 - 13|$ **8.** $|39{,}000|$

Write < or > in the blank.

9. -8 ____ 3 **10.** 54.6 ____ 39.897 **11.** $-\frac{1}{6}$ ____ $-\frac{5}{6}$

12. $7\frac{1}{4}$ ____ $5\frac{8}{9}$ **13.** -20.876 ____ -20.8759 **14.** $-\frac{7}{16}$ ____ $-\frac{37}{79}$

15. -15 ____ -17 **16.** -0.05 ____ 0.05 **17.** $|-45|$ ____ 46

18. -20 ____ $|-19|$ **19.** $|74 - 32|$ ____ 33 **20.** 65 ____ $|-64|$

(Answers on page 338)

ACT-TYPE PROBLEMS

1. The symbol > could be used to make all of the following true EXCEPT:

A. 45 ____ -56
B. $|-35|$ ____ -36
C. -34 ____ 2
D. $|-15|$ ____ 4
E. 5 ____ -6

2. Given the following:

I. $|-3| < 2$
II. $-6 > -1$
III. $14 > -16$

Which of the following choices contain all true statements?

F. I
G. III
H. I and III
J. II and III
K. I, II, and III

3. $5 + |-3| - 8 + 3 = ?$

A. -3
B. -1
C. 0
D. 1
E. 3

4. Which of the following choices lists the numbers $\{-3, |-9|, 4, -5, |-10|\}$ in order from least to greatest?

F. $-3, 4, -5, |-9|, |-10|$
G. $4, -3, -5, |-10|, |-9|$
H. $|-10|, |-9|, -5, -3, 4$
J. $-5, -3, 4, |-9|, |-10|$
K. $-5, |-9|, |-10|, -3, 4$

5. Which of the following is the largest number?

A. -110
B. $|-120|$
C. 105
D. -200
E. $|105|$

(Answers on page 338)

Computation With Positive and Negative Numbers

CALCULATOR TIP

Use your calculator to complete or to check computations with positive and negative numbers. Remember, a calculator uses the [−] key for subtraction and the [+/−] or [(−)] key to represent a negative number. Be sure to use these keys correctly. Enter [+/−] 7 or [(−)] 7 and the calculator shows −7.

Chapter 7 ▪

Pre-Algebra II

- Fourteen ACT questions have to do with pre-algebra.
- Easier pre-algebra questions may be about a single skill or concept.
- More difficult questions will often test a combination of skills or concepts.
- The pre-algebra review in Chapters 6 and 7 covers all the material you need to answer ACT questions.
- Use a calculator for the ACT-Type Problems. Do not use a calculator for the Practice exercises.

▪ Percent

Percent means per one hundred or out of one hundred.

15% means 15 out of 100.

1.5% means 1.5 out of 100.

Decimals, percents, and fractions can all be used to name the same number.

- To write a decimal as a percent, move the decimal point two places to the right and write the percent sign.

  0.78 = 78%   0.09 = 9%   0.0524 = 5.24%   28.634 = 2,863.4%

- To write a percent as a decimal, move the decimal point two places to the left. Write zeros if necessary.

  36% = 0.36   7% = 0.07   0.034% = 0.00034   386.29% = 3.8629

- To write a fraction as a decimal, divide the numerator by the denominator. To change this decimal to a percent, follow the rule above.

Write $\frac{3}{8}$ as a percent.

```
  0.375 = 37.5%
8)3.000
  2 4
    60
    56
     40
     40
      0
```

Write $\frac{2}{3}$ as a percent.

$3\overline{)2.00}$ = 0.6666... = $66.\overline{6}\%$

The decimal for $\frac{2}{3}$ is a repeating decimal. The digit 6 repeats.

You can write $66.\overline{6}\%$ as $66\frac{2}{3}\%$.

- To write a percent as a fraction, write the percent as a fraction with 100 in the denominator. Then simplify the fraction if possible.

Write 56% as a fraction.

$$56\% = \frac{56}{100} = \frac{14}{25}$$

Write 0.84% as a fraction.

$$0.84\% = \frac{0.84}{100} = \frac{84}{10{,}000} = \frac{21}{2{,}500}$$

It will help you to know the fraction and percent equivalents in the following table.

Fractions and Percents

$\frac{1}{4} = 25\%$	$\frac{1}{2} = 50\%$	$\frac{3}{4} = 75\%$	
$\frac{1}{5} = 20\%$	$\frac{2}{5} = 40\%$	$\frac{3}{5} = 60\%$	$\frac{4}{5} = 80\%$
$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{5}{6} = 83\frac{1}{3}\%$
$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$

CALCULATOR TIP

Use a calculator to convert among percents, decimals, and fractions. Many calculators have special keys for this.

MODEL ACT PROBLEM

Lucy made 9 out of 15 basketball free-throw shots. Ann made 18 of 25 basketball free-throw shots. What is the difference between Lucy's free-throw percentage and Ann's free-throw percentage?

A. 9%
B. 12%
C. 18%
D. 25%
E. 60%

SOLUTION

Use a calculator. Find each free-throw percentage by changing the fraction to a decimal and writing the decimal as a percent.

$\frac{9}{15} = 0.60 = 60\%$ $\frac{18}{25} = 0.72 = 72\%$

Subtract. 72% − 60% = 12%

The correct answer is B.

Practice

Complete the table. The three numbers across in each row should be equal.

Fraction	*Decimal*	*Percent*
1.	2.	18%
3.	0.036	4.
$\frac{5}{8}$	5.	6.
7.	8.	0.84%
9.	0.0002	10.
$\frac{5}{6}$	11.	12.

13. Jennifer ran $\frac{3}{8}$ of a mile. What percent of a mile did she run?
14. Bob is 75% of Chad's height. What fraction of Chad's height is Bob?
15. Jim's gas tank was $\frac{1}{4}$ full. What percent of his gas tank was full?
16. Frank wins 30% of the time. What fraction of the time does Frank win?
17. Dave rides his bike to work $\frac{2}{3}$ of the time. What percent of the time does Dave ride his bike to work?
18. Eric works out 5 days a week and he jogs on 4 of those days. On what percent of the workout days does he jog?
19. Rachel has 6 dresses and 50% of them are red. What fraction of the dresses are red?
20. Matt makes 80% of the money that Andy makes. What fraction of Andy's money does Matt make?

(Answers on page 340)

ACT-TYPE PROBLEMS

1. Leticia works in the personnel department at a company. Two out of every five people she interviews are female. What percent of the people that Leticia interviews are female?

   A. 20%
   B. 30%
   C. 40%
   D. 50%
   E. 60%

2. Of the people who went to see a movie, 65% were over 18. What fraction of the people who went to see the movie were under 18?

   F. $\frac{7}{10}$
   G. $\frac{13}{20}$
   H. $\frac{6}{13}$
   J. $\frac{7}{20}$
   K. $\frac{3}{10}$

3. Francine cooks vegetable stir-fry 15 days in 4 weeks and chicken stir-fry 6 days in 4 weeks. What percent of the days does Francine cook stir-fry?

   A. 25%
   B. 40%
   C. 50%
   D. 70%
   E. 75%

4. On one math test Beth got 84% of the questions correct. On another test she got 6 of 15 answers correct. What is the difference between the fraction of questions Beth answered correctly on the two tests?

   F. $\frac{2}{5}$
   G. $\frac{11}{25}$
   H. $\frac{13}{25}$
   J. $\frac{3}{5}$
   K. $\frac{32}{75}$

5. There are 8 chairs around one dining room table. Three of the chairs have arm-rests. There are 8 chairs around another table, and 7 of them have arm-rests. What percent of the 16 chairs have arm-rests?

   A. 47.5%
   B. 50%
   C. 55%
   D. 62.5%
   E. 70%

(Answers on page 340)

Percent Problems

The basic relationship for percent problems is

Percent × base = percentage

25% of 80 = 20

In a percent problem you need to find one of the three quantities.

Finding a Percent of a Number

Write an equation and solve to find the percentage.

EXAMPLES

1. Bob drove 30% of his 235-mile trip. How far is that?

   Decide which quantity is missing. $0.3 \times 235 = ?$

   To find the percent, multiply. $0.3 \times 235 = 70.5$

   Bob drove 70.5 miles.

2. Liz saved $33\frac{1}{3}\%$ off the list price of $96. How much did she save?

   Recall that $33\frac{1}{3}\% = \frac{1}{3}$ $\quad \frac{1}{3} \times \$96 = ?$ $\quad \frac{1}{3} \times 96 = 32$

   Liz saved $32.

Finding the Percent One Number Is of Another

Write an equation and solve to find the percent.

EXAMPLES

1. Ben completed 6 miles of a 16-mile trip. What percent of the trip is completed?

   $? \times 16 = 6$ $\qquad$ percent $= \frac{?}{100}$

   $\frac{n}{100} \times 16 = 6$

   $6 \div 16 = 0.375$. Write 37.5 as a percent.

   Ben completed 37.5% of the trip.

2. The wholesale price of a ring was \$28. A jeweler sold it for \$42. What percent of the wholesale price was the selling price?

   $? \times 28 = 42$

   $42 \div 28 = 1.5 = 150\%$

   The selling price was 150% of the wholesale price.

The base is not always the larger number. The percent may be greater than 100%.

Finding a Number When a Percent of It Is Known

Write an equation and solve to find the base.

EXAMPLE

When it is 30% full a container holds 6 gallons. How much does the container hold?

30% of $? = 6$ $\qquad$ $30\% \times ? = 6$

$6 \div 30\% = 6 \div 0.3 = 20$

The container holds 20 gallons.

Sales Tax and Discount

Everyday transactions often involve percents. A sales tax adds a certain percentage of the cost of an item to the price you have to pay. A discount reduces the price of an item by a percentage of its original cost.

Sales Tax

EXAMPLE

Aaron bought a new television for \$1,199. What is the total cost of the television including 6% sales tax?

$6\% \times \$1,199 = 0.06 \times \$1,199$

$0.06 \times \$1,199 = \$71.94$

$\$71.94 + \$1,199 = \$1,270.94$

The total cost of the television is \$1,270.94.

You can use either a decimal or a fraction for a percent.

Discount

EXAMPLE

Kerine saved 25% off the list price of \$96. How much did she pay?

25% of $\$96 = ?$ $\qquad$ $25\% = \frac{1}{4}$

$\frac{1}{4} \times 96 = 24$

$\$96 - \$24 = \$72$

Kerine paid \$72.

Percent of Increase or Decrease

Use these formulas to find the percent of increase and the percent of decrease.

$$\text{Percent of increase} = \frac{\text{New amount} - \text{Original amount}}{\text{Original amount}}$$

$$\text{Percent of decrease} = \frac{\text{Original amount} - \text{New amount}}{\text{Original amount}}$$

Write the result as a percent.

CALCULATOR TIP

Most calculators have a fully functional percent key. This key calculates the amount of decrease or increase (discount or tax) and the final price including tax and/or discount.

Percent of Increase

EXAMPLE

The price of an item increases from \$30 to \$34.80. Find the percent of increase.

1. Subtract to find the amount of increase. The amount of increase is \$4.80. $\qquad$ $\$34.80 - \$30 = \$4.80$

2. Divide the amount of increase by the original amount. $\qquad$ $\frac{\text{Amount of increase}}{\text{Original amount}} \rightarrow \frac{\$4.80}{\$30}$

3. Write as a percent. $\qquad$ $30\overline{)4.8} = 0.16 = 16\%$

   The percent of increase is 16%.

Percent of Decrease

EXAMPLE

The price of an item decreases from $25 to $21.75. Find the percent of decrease.

1. Subtract to find the amount of decrease. The amount of decrease is $3.25.

   $\$25 - \$21.75 = \$3.25$

2. Divide the amount of decrease by the original amount.

   $\frac{\text{Amount of decrease}}{\text{Original amount}} \rightarrow \frac{\$3.25}{\$25}$

   $3.25 \div 25 = 0.13 = 13\%$

3. Write as a percent.

   The percent of decrease is 13%.

MODEL ACT PROBLEMS

1. Steve stopped for a drink of water when he had completed 60% of his jog. He had traveled 3 miles. What is the total distance Steve jogged?

   A. 2 miles
   B. 3 miles
   C. 4 miles
   D. 5 miles
   E. 6 miles

SOLUTION

Use the equation: percent × base = percentage

$0.6 \times \text{base} = 3 \text{ miles}$

$\text{base} = \frac{3}{0.6} = 5 \text{ miles}$

Steve jogged 5 miles.

The correct answer is D.

2. Cassandra paid $35 for a shirt that originally cost $50. What percent of the original price did the shirt cost?

   F. 40%
   G. 50%
   H. 65%
   J. 70%
   K. 85%

SOLUTION

The percentage is the price after the sale ($35). The base is the original price of the shirt ($50).

percent × base = percentage

percent × 50 = 35

$\text{percent} = \frac{35}{50} = 0.70 = 70\%$

Cassandra paid 70% of the original price for the shirt.

The correct answer is J.

3. Jeff bought a car originally priced at $10,700. However, there was a sale so he got the car for $9,630. What was the percent of decrease?

   A. 10%
   B. 20%
   C. 30%
   D. 40%
   E. 50%

SOLUTION

Use the equation:

$$\text{percent of decrease} = \frac{\text{Original amount} - \text{New amount}}{\text{Original amount}}$$

$$\text{percent of decrease} = \frac{\$10{,}700 - \$9{,}630}{\$10{,}700} = \frac{\$1{,}070}{\$10{,}700} = 10\%$$

The percent of decrease is 10%.

The correct answer is A.

4. Tessa bought a car that cost $15,800. What was the total cost of the car including an 8% sales tax?

   F. $14,536
   G. $15,800
   H. $17,064
   J. $17,500
   K. $18,000

SOLUTION

Find the 8% sales tax.

Use the equation:

percent × base = percentage

$0.08 \times \$15{,}800 = \$1{,}264$

Add the sales tax to the price of the car.

$\$15{,}800 + \$1{,}264 = \$17{,}064$

The total cost of the car was $17,064.

The correct answer is H.

Practice

1. What is the cost of a $99 item selling for 75% of that price?
2. A tank is at $62\frac{1}{2}\%$ of its 640-gallon capacity. How many gallons are in the tank?
3. An acorn grew 0.5% from its weight of 2.5 grams. How much did it grow?
4. After two weeks on a diet Raymond weighed 180 pounds, which was 90% of his original weight. What was Raymond's original weight?
5. What percent of a 120-gallon tank is full if it contains 90 gallons?
6. A 39-foot pole casts a 26-foot shadow. What percent of the pole's height is the shadow's length?
7. Nine out of 60 cans of dog food have been eaten. What percent of the dog food has been eaten?
8. Chris is 20 years old and Susan is 50 years old. What percent of Susan's age is Chris?
9. When 19% full, a tank contains 136.8 gallons. How much does the full tank hold?
10. When a cup is 75% filled it contains 75 ounces. How much can the cup hold?
11. A certain baseball stadium is 60% filled when it has 15,000 people in attendance. How many people can the stadium hold?
12. A price increased from $15 to $18.30. What is the percent of increase?
13. An $82 item is on sale for $50.84. What is the percent of decrease?
14. The number of students in a school decreased from 4,850 to 3,104. What is the percent of decrease?
15. A balloon went from 13,500 feet to 19,008 feet. What is the percent of increase?
16. What is the total cost of a $28.50 garden rake including an 8% sales tax?

During the End-of-Summer Sale, an air conditioner that originally sold for $510 was discounted 25%.

17. How much would you save if you waited for this sale?
18. What is the sale price of the air conditioner?

Phil bought a computer for $2,000.

19. What is the final cost of Phil's computer including a 6.5% sales tax?
20. How much money would Phil save if he did not have to pay an 8% shipping charge? (The shipping charge is applied before the tax.)

(Answers on page 341)

ACT-TYPE PROBLEMS

1. 21 is 40% of which of the following numbers?

   A. 40
   B. 46.5
   C. 50.2
   D. 52.5
   E. 55

2. Mylin's Magic Club collects $4,000 from the sale of tickets when there is a full house. If all the tickets cost the same amount, how much money will Mylin collect from the purchase of tickets if the club is 75% full?

   F. $2,000
   G. $2,500
   H. $3,000
   J. $3,250
   K. $3,500

3. Joe has to pay $600 rent. He has only 80% of the money. If Joe takes home $6 per hour, how many hours must he work to collect the rest of the rent money?

A. 10 hours
B. 15 hours
C. 20 hours
D. 25 hours
E. 30 hours

4. Ellen walks 0.75 mile to school each day. She stops to get breakfast after completing 60% of her walk. How far has Ellen walked when she stops for breakfast?

F. 0.35 mile
G. 0.45 mile
H. 0.50 mile
J. 0.55 mile
K. 0.65 mile

5. Dan plays basketball on his high school team. During the last game Dan scored 18 points, while for the season he averages 20 points per game. What percent of Dan's average did he score during this game?

A. 60%
B. 65%
C. 75%
D. 80%
E. 90%

6. During a soccer game Chad scored 2 of his team's 6 goals. What percent of his team's goals did Chad score?

F. 33%
G. $33\frac{1}{3}\%$
H. 66%
J. $66\frac{2}{3}\%$
K. 77%

7. Shannon bought a house for $150,000 and then she had to pay a 9% tax. What was the total cost of the house including the tax?

A. $160,000
B. $162,500
C. $163,500
D. $165,000
E. $165,500

8. Emma found a barbecue grill she wanted that originally cost $120. She discovered it was on sale for 10% off the original price. Emma bought the grill on sale and paid a 6% sales tax. What was the final cost of the grill?

F. $112.54
G. $113.50
H. $114.48
J. $115.86
K. $116.35

9. The original price of a television was $225. During a sale the price went down to $180. What was the percent of decrease?

A. 10%
B. 15%
C. 20%
D. 25%
E. 30%

10. In one month Dennis' puppy grew from 3 pounds to 5 pounds. What was the percent of increase?

F. 22%
G. $22\frac{1}{2}\%$
H. 33%
J. 66%
K. $66\frac{2}{3}\%$

(Answers on page 341)

Ratio and Proportion

A ratio can be expressed in three ways.

5 to 6 $\quad \frac{5}{6} \quad$ 5:6

Proportions

A **proportion** shows that two ratios are equal. If you write the ratios as fractions, the cross products are equal.

$$11{,}063 \quad \frac{23}{37} \times \frac{299}{481} \quad 11{,}063$$

Since the cross products are equal, these two fractions form a proportion.

CALCULATOR TIP

Some calculators solve proportions directly. You can always use a calculator to complete or to check your calculations.

Solving a Proportion

If one of the values in the proportion is unknown, cross multiply to solve the proportion.

EXAMPLES

1. Two fractions are equivalent. One of the fractions is $\frac{13}{15}$ and the numerator of the second fraction is 156. Solve a proportion to find the denominator of the second fraction.

First, write a proportion.

first fraction → $\frac{13}{15} = \frac{156}{x}$ ← second fraction

Solve the proportion.

Cross multiply.	Cross products are equal.	Divide to find x.
$13x \quad \frac{13}{15} \times \frac{156}{x} \quad 2{,}340$	$13x = 2{,}340$	$x = 2{,}340 \div 13$ $x = 180$

The denominator of the second fraction is 180.

2. A builder estimates that she will use 20 bricks to cover 3 square feet of wall. How many square feet can be covered with 2,090 bricks?

Write a proportion. $\frac{20 \text{ bricks}}{3 \text{ square feet}} = \frac{2{,}090 \text{ bricks}}{x \text{ square feet}}$

Solve the proportion.

Cross multiply.	Cross products are equal.	Divide to find x.
$20x \quad \frac{20}{3} \times \frac{2{,}090}{x} \quad 6{,}270$	$20x = 6{,}270$	$x = 6{,}270 \div 20$ $x = 313.5$

The builder can cover 313.5 square feet with 2,090 bricks.

When writing a proportion, be sure to write each ratio in the same way. Here we wrote each ratio as bricks to square feet.

3. A 6-foot-tall person casts an 8.4-foot shadow. A telephone pole right next to the person casts a 20.3-foot shadow. How tall is the telephone pole?

Write a proportion.

person → $\frac{\text{6-foot-tall person}}{\text{8.4-foot shadow}} = \frac{x\text{-foot-tall pole}}{\text{20.3-foot shadow}}$ ← telephone pole

Solve the proportion.

Cross multiply.	Cross products are equal.	Divide to find x.
$\frac{6}{8.4} \bowtie \frac{x}{20.3}$	$8.4x = 121.8$	$x = 121.8 \div 8.4$
		$x = 14.5$

The telephone pole is 14.5 feet high.

The proportion could also be written $\frac{8.4}{6} = \frac{20.3}{x}$. The result will be the same.

MODEL ACT PROBLEM

1. Ed is a baseball player who gets a hit 6 times out of every 20 times at bat. How many hits will Ed expect to get if he is at bat 200 times?

A. 50 hits
B. 55 hits
C. 60 hits
D. 65 hits
E. 70 hits

SOLUTION

Set up a proportion.

$\frac{\text{6 hits}}{\text{20 times at bat}} = \frac{x \text{ hits}}{\text{200 times at bat}}$

Cross multiply and solve for x.

$6 \times 200 = 20x$

$1{,}200 = 20x$

$60 = x$

Ed would expect to have 60 hits after 200 times at bat.

The correct answer is C.

2. In two weeks the Postman family drinks 3 gallons of milk. How many gallons of milk will the Postmans drink in 9 weeks?

F. 3 gallons
G. 6 gallons
H. 13.5 gallons
J. 18 gallons
K. 27 gallons

SOLUTION

Set up a proportion.

$\frac{\text{3 gallons}}{\text{2 weeks}} = \frac{x \text{ gallons}}{\text{9 weeks}}$

Cross multiply and solve for x.

$3 \times 9 = 2x$

$27 = 2x$

$x = 13.5$

The Postmans will drink 13.5 gallons of milk in 9 weeks.

The correct answer is H.

Practice

Rewrite each ratio in two different ways.

1. $\frac{5}{7}$
2. 3:13
3. 4 to 9
4. $\frac{2}{5}$

Solve the proportion.

5. $\frac{2}{3} = \frac{x}{9}$
6. $\frac{x}{5} = \frac{3}{25}$
7. $\frac{4}{x} = \frac{10}{13}$
8. $\frac{3}{7} = \frac{x}{49}$
9. $\frac{4}{8} = \frac{7}{x}$
10. $\frac{1}{5} = \frac{x}{100}$
11. $\frac{5}{11} = \frac{x}{121}$
12. $\frac{12}{x} = \frac{66}{27.5}$

13. Solve the proportion $\frac{5}{13} = \frac{20}{x}$.
14. Show $\frac{4}{9}$ written as a ratio in two other ways.
15. It costs David $25 to fill up his tank of gas twice. How much will it cost David to fill up his tank of gas 20 times?
16. Samantha has two cats, Cal and Hob. For every scoop of food that Hob eats, Cal eats 3 scoops. If Hob eats ten scoops of food, how many scoops of food will Cal eat?
17. You get 12 eggs for every carton you buy. How many eggs will you get if you buy 7 cartons?
18. A roofer estimates that 15 tiles are needed to cover 4 square feet. How many tiles will be needed to cover 120 square feet of the same roof?
19. A flagpole that stands 25 feet high casts a shadow of 40 feet. Someone standing right next to the flagpole is 6 feet tall. How long is the person's shadow?
20. Jack went to the grocery store and bought 8 apples for $2. How many apples could Jack have bought if he had $5?

(Answers on page 342)

ACT-TYPE PROBLEMS

1. John is making tacos for himself and 17 friends. For every 3 people John uses 5 tomatoes. How many tomatoes will John use in all?

A. $10\frac{1}{5}$ tomatoes
B. $10\frac{4}{5}$ tomatoes
C. $28\frac{1}{3}$ tomatoes
D. 30 tomatoes
E. 85 tomatoes

2. Some say that a person should drink 8 glasses of water a day. If each of 45 people follows this advice, how much water will they drink?

F. 300 glasses of water
G. 340 glasses of water
H. 350 glasses of water
J. 360 glasses of water
K. 390 glasses of water

3. Given the ratio $\frac{5}{6}$.

I. 5 to 6
II. 6:5
III. 5:6

Which of the following choices is a complete list of the ways to properly rewrite the ratio?

A. I
B. II
C. II and III
D. I and III
E. I, II, and III

4. Jeff went to a book sale where 6 books cost $13. How many books could Jeff buy at that rate for $32.50?

F. 10 books
G. 15 books
H. 17 books
J. 19 books
K. 23 books

5. A trainer uses 0.75 roll of medical tape on each soccer player. If there are 15 players on the team and 80% get taped up, how many rolls of tape does the trainer need?

A. 8 rolls of tape
B. 9 rolls of tape
C. 10 rolls of tape
D. 11 rolls of tape
E. 12 rolls of tape

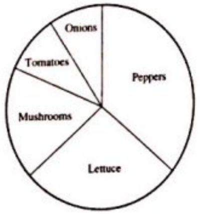
(Answers on page 342)

3. Alex likes to eat salads for lunch. Alex makes his salads with lettuce, tomatoes, onions, mushrooms, and peppers. Alex uses 2 times as many mushrooms as peppers, the same amount of onions as peppers, 3 times as many tomatoes as peppers, and 4 times as much lettuce as peppers. Which of the following circle graphs best fits this information?

A.

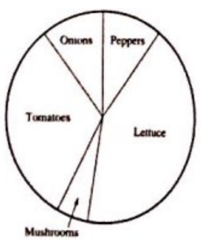
B.

C.



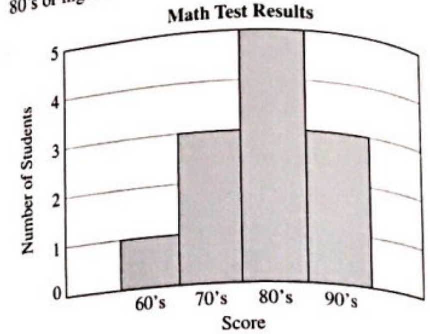
D.

E.



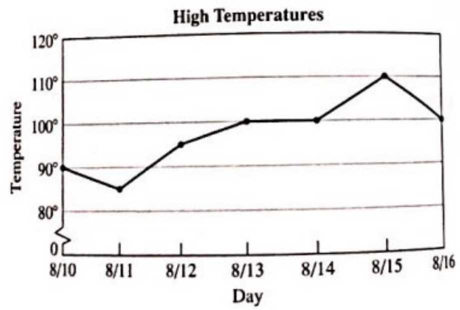
(Answers on page 344)

4. The bar graph below shows the scores on a recent math test. What percentage of the class scored in the 80's or higher?



F. 8%
G. 25%
H. $41\frac{2}{3}\%$
J. $66\frac{2}{3}\%$
K. $91\frac{2}{3}\%$

5. During an August heat wave, Wendy kept track of the high temperature every day for a week. She graphed her results on a line graph. To the nearest degree, what was the average high temperature that week?



A. 95°
B. 96°
C. 97°
D. 98°
E. 100°

Probability

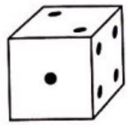
The probability of an event is the likelihood that it will occur. If an event will never occur, the probability is 0. If an event will always occur, the probability is 1. All other probabilities fall between 0 and 1. Use a fraction to write the probability of an event.

$$\text{Probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLES

Think of rolling a single fair die.

1. What is the probability of rolling a 3?



There are six sides and each side is equally likely to land faceup.

$$P(3) = \frac{\text{number of sides with a 3}}{\text{number of sides in all}} \begin{matrix}\to 1 \\ \to 6\end{matrix}$$

The probability of rolling a 3 is $\frac{1}{6}$.

2. What is the probability of rolling an even number?

$$P(\text{even number}) = \frac{\text{number of sides with an even number (2, 4, 6)}}{\text{number of sides in all}} \begin{matrix}\to 3 \\ \to 6\end{matrix} = \frac{1}{2}$$

The probability of rolling an even number is $\frac{1}{2}$.

3. What is the probability of rolling a 7?

$$P(7) = \frac{\text{number of sides with a 7}}{\text{number of sides in all}} \begin{matrix}\to 0 \\ \to 6\end{matrix} = 0$$

The probability of rolling a 7 is 0. This means that the event is impossible.

4. What is the probability of rolling a number less than 10?

$$P(N < 10) = \frac{\text{number of sides with a number less than 10}}{\text{number of sides in all}} \begin{matrix}\to 6 \\ \to 6\end{matrix} = 1$$

The probability of rolling a number less than 10 is 1. This means that the event is certain to happen.

Independent and Dependent Events

Independent events — The outcome of one event *does not* affect the probability of the other event.

EXAMPLE

You have a standard deck of 52 cards. The probability of picking the king of hearts is $\frac{1}{52}$. You pick a card without looking and replace it in the deck. There are still 52 cards in the deck. The probability of picking the king of hearts is still $\frac{1}{52}$. A card picked and replaced followed by picking another card are independent events.

Dependent events — The outcome of one event *does* affect the probability of the other event.

EXAMPLE

You have a standard deck of 52 cards. The probability of picking the king of hearts is $\frac{1}{52}$. You pick a card without looking, not the king of hearts, and don't put it back. There are now 51 cards in the deck. The probability of picking the king of hearts is now $\frac{1}{51}$. A card picked and not replaced followed by picking another card are dependent events.

MODEL ACT PROBLEMS

1. In a jar there are 3 blue marbles and 5 red marbles. If you reach into the jar and pick out a marble without looking, what is the probability that you will pick out a blue marble?

A. $\frac{3}{8}$

B. $\frac{3}{5}$

C. $\frac{5}{8}$

D. $\frac{8}{5}$

E. $\frac{8}{3}$

SOLUTION

$$P(\text{blue marble}) = \frac{\text{number of blue marbles}}{\text{total number of marbles}} = \frac{3}{8}$$

The probability of picking a blue marble is $\frac{3}{8}$.

The correct answer is A.

2. What is the probability of choosing, without looking, a red face card from a regular deck of cards? (A face card is a jack, a queen, or a king.)

F. $\frac{1}{26}$

G. $\frac{1}{13}$

H. $\frac{3}{26}$

J. $\frac{1}{4}$

K. $\frac{1}{2}$

SOLUTION

There are 6 red face cards in a deck (There are 4 kings, 4 queens, and 4 jacks, or a total of 12 face cards. Half of the face cards are red.)

Find the probability of getting a red face card.

$$P(\text{red face card}) = \frac{\text{number of red face cards}}{\text{total number of cards}}$$

$$= \frac{6}{52} = \frac{3}{26}$$

The probability of choosing a red face card from the deck is $\frac{3}{26}$.

The correct answer is H.

Practice

You have a fair penny.

1. What is the probability of flipping a tail?

2. What is the probability of flipping a head?

3. What is the probability of flipping a head or a tail?

4. You flip the penny 10 times and get 10 heads. What is the probability of getting a head on the next flip?

You have six same-size balls numbered 1, 2, 4, 5, 7, and 8 in a box. You pick one without looking. What is the probability of picking

5. an 8?

6. an even number?

7. a number greater than 6?

8. a number divisible by 6?

9. a multiple of 4?

You have a standard deck of 52 cards. What is the probability of picking

10. a 7?

11. a red card?

12. a heart?

13. a non-face card?

14. a 2, 3 , or 4?

15. a 9 or 10?

16. a jack, or a queen, or a king?

You pick a 7 of clubs and don't replace it. What is the probability that the next pick will be

17. a 6 of clubs?

18. a 7 of clubs?

19. a 7?

20. a club?

(Answers on page 344)

11. 12. 13. 14

ACT-TYPE PROBLEMS

1. In a drawer there are 3 brown socks, 2 blue socks, and 5 red socks. You pick a sock without looking. What is the probability of choosing a brown sock?
   - A. $\frac{2}{3}$
   - B. $\frac{3}{10}$
   - C. $\frac{1}{2}$
   - D. $\frac{1}{5}$
   - E. $\frac{3}{7}$

2. What is the probability of rolling a 1 or a 5 with a single six-sided die?
   - F. $\frac{1}{6}$
   - G. $\frac{1}{3}$
   - H. $\frac{1}{2}$
   - J. $\frac{2}{3}$
   - K. $\frac{5}{6}$

3. From a regular deck of 52 cards, what is the probability of choosing a card that is not an ace or a face card, but that is divisible by 2?
   - A. $\frac{3}{26}$
   - B. $\frac{1}{13}$
   - C. $\frac{1}{4}$
   - D. $\frac{7}{13}$
   - E. $\frac{5}{13}$

4. If a king and a jack are removed from a regular 52-card deck, what is the probability of picking a face card?
   - F. $\frac{1}{5}$
   - G. $\frac{1}{3}$
   - H. $\frac{2}{3}$
   - J. $\frac{2}{5}$
   - K. $\frac{3}{5}$

5. There are 20 colored pencils in a box: 4 red, 5 green, 10 blue, and 1 black. You pick a pencil without looking. What is the sum of the probability of picking a red pencil and the probability of picking a black pencil?
   - A. $\frac{7}{10}$
   - B. $\frac{1}{5}$
   - C. $\frac{1}{20}$
   - D. $\frac{1}{4}$
   - E. $\frac{1}{2}$

(Answers on page 344)

Elementary Counting Techniques

There are some elementary techniques that can help you to count very efficiently. This section reviews three of these techniques: products, permutations, and combinations. Factorials are used to find the number of permutations or combinations.

CALCULATOR TIP

Many calculators have a way to compute factorials, permutations, and combinations. Be sure to use these functions on your calculator.

Products

You may just need to multiply. Look at the example below.

EXAMPLES

You are buying a frozen yogurt cone. The yogurt store has three different types of cones, six different flavors, and eight kinds of toppings. Multiply to find how many types of yogurt cones you can buy.

3	×	6	×	8	=	144
cones		flavors		toppings		

There are 144 different types of cones.

Permutations

A permutation is the arrangement of a certain number of items in a specific order.

EXAMPLES

Three students, Alex, Bonnie, and Charles, line up single file. How many different ways can they line up? You can make a list.

A B C	B A C	C A B
A C B	B C A	C B A

Three students can line up single file six different ways.

You can also use **factorial** to find the number of different permutations.
3 factorial is written 3!. 3! means $3 \times 2 \times 1$, or 6.

Use 6 factorial to find the number of different permutations of 6 items.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

There are 720 different ways to arrange six items.

According to a special rule, $0! = 1$.

EXAMPLE

Nine students are going to line up single file for movie tickets. In how many different ways can the students line up?

Use $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362{,}880$

There are 362,880 ways for the students to line up.

Permutation Formula

You can always use this formula to find the number of permutations, or ways of arranging n items in r positions.

$$\frac{n!}{(n-r)!}$$

Look at the examples on the next page.

9, 10

EXAMPLES

1. Four students in a club are interested in two positions, president and vice president. How many ways can the students be chosen for the two positions?

   Use the permutation formula.

   There are four students and two different positions.

   So $n = 4$ and $r = 2$. Substitute.

   $$\frac{n!}{(n-r)!} = \frac{4!}{2!} = \frac{24}{2} = 12$$

   There are 12 ways of electing four students to two positions.

2. There are five cars and three parking spaces. In how many different ways can the cars be parked in the spaces?

   $n = 5$ and $r = 3$

   $$\frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$$

   There are 60 ways to park 5 cars in three spaces.

Order matters here. First—you have 4 to choose from. Second—you have only 3 left to choose from. So $4 \times 3 = 12$ ways.

Combinations

A combination is an arrangement of a certain number of items in which order does not matter. Look at this example.

EXAMPLE

There are just two parking spaces, but four cars—red, blue, yellow, and green. In how many different ways can two cars be parked? Order doesn't matter.

We are taking four things, two at a time.

Make a list. Remember, order doesn't matter.

RB	BY	YG
RY	BG	
RG		

There are six ways of parking four cars in two spaces.

Order doesn't matter here. There are fewer ways.

$$\frac{4 \text{ (choices)}}{1 \text{ (1st pick)}} \times \frac{3 \text{ (choices)}}{2 \text{ (2nd pick)}}$$

$$\frac{4 \times 3}{1 \times 2} = \frac{12}{2} = 6 \text{ ways}$$

Combination Formula

You can always use this formula to find the number of combinations that can be made from a group of n items taken r at a time.

$$\frac{n!}{(n-r)!r!}$$

Since we are taking four things two at a time, $n = 4$ and $r = 2$.

Substitute.

$$\frac{n!}{(n-r)!r!} = \frac{4!}{(4-2)!(2)!} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 2} = \frac{24}{4} = 6$$

There are six ways of parking the cars if order does not matter.

MODEL ACT PROBLEM

There are three job openings for Level I computer technicians, and six applicants. How many different ways could these six applicants be chosen to fill the three job openings?

A. 6
B. 20
C. 120
D. 520
E. 720

SOLUTION

The problem asks about choosing, not ranking, applicants. Order does not matter, so this is a combination problem.

$$\frac{n!}{(n-r)!(r)!} = \frac{6!}{(6-3)!(3)!}$$

$$= \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{3 \times 2 \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}} = \frac{120}{6} = 20$$

The correct answer is B.

Practice

1. How many different ways are there to place 5 books on 5 different shelves if you can place only one book on each shelf?
2. If Andy has 7 shirts, 6 pants, and 8 ties, how many different outfits can he make with the pants, shirts, and ties?
3. There are 4 coach seats left on an airplane and 7 people waiting for those seats. How many different ways can you choose people to fill the 4 seats?
4. There are 4 people running for 4 different positions: President, Vice President, Secretary, and Treasurer. How many different ways are there to place the 4 people into these 4 different positions?
5. For breakfast Jane has cereal, orange juice, an apple, and a piece of toast. Jane keeps 4 different types of cereal, 2 different kinds of orange juice, 3 different types of apples, and 2 types of bread. How many different breakfast choices does she have?
6. There are 6 puppies born, but the mother always feeds only 4 puppies at a time. How many different groups of puppies can she feed at one time?
7. There are 5 windows in a room, and Rich bought 5 different curtains to go on the windows. How many different ways can Rich place the curtains on the windows?
8. Steve knows 5 notes on the guitar and plays 4 notes in a row. How many different ways will Steve be able to arrange his notes?
9. There are 9 different things to drink in Lisa's house, but Lisa has only 3 glasses. How many different ways are there to put 3 different drinks in the glasses. (You can put only one type of drink into each glass.)
10. A 7-digit phone number uses all the digits from 3 to 9. How many different possible phone numbers are there?

(Answers on page 344)

ACT-TYPE PROBLEMS

1. Eight students try out for two openings on the debate team. In how many different ways can these two openings be filled?

   A. 2
   B. 8
   C. 28
   D. 56
   E. 84

2. A store manager has four different gifts to give to the first four people who enter the store. In how many different ways can he distribute the gifts?

   F. 4
   G. 8
   H. 16
   J. 24
   K. 32

3. A license plate has three letters (A–Z) followed by three digits (0–9). How many different license plates can be produced?

   A. 11,232,000
   B. 12,654,720
   C. 12,812,904
   D. 15,600,000
   E. 17,576,000

4. A newspaper editor needs five stories for the front page of the next issue. There are six reporters available to write the stories. How many ways can they be assigned if each story is written by only one reporter?

   F. 5
   G. 6
   H. 30
   J. 120
   K. 720

5. How many 3-digit area codes can be created if the first digit cannot be zero?

   A. 100
   B. 300
   C. 729
   D. 900
   E. 999

(Answers on page 344)

Writing Linear Expressions and Equations

You may have to write an expression or an equation to solve a problem. An **equation** is a statement about variables and numbers that contains an equal sign. An **expression** contains no equal sign. The variables in a **linear equation** are of the first degree. (They have exponents of 1.)

Words or phrases in the problem may help you decide when to write an operation sign or when to write an equal sign. But remember that you can't just use these words and phrases without thinking.

Operation	Words and Phrases
addition	and, more, in all, increased by, sum, total
subtraction	less, decreased by, difference, how many more/less
multiplication	of, product, times
division	per, quotient, shared
equals	is, equal to, equals

EXAMPLES

1. Write an expression for this statement.

   Three times the sum of x and y.

   Substitute symbols and operation signs for the words. $3 \times (x + y)$

2. Write an equation for this problem.

   The Rooster baseball team scored 8 runs in the first four innings. The Stars scored 3 runs in the first four innings. After four innings, how many more runs have the Roosters scored?

   The variable in this problem is the difference in the number of runs each team has scored. Let d equal the difference.

   Write the problem in words. The difference equals 8 less 3.

   Write an equation. $d = 8 - 3$

3. Write an equation for this problem.

   A magnolia tree on Kiefer's property is 7 feet 3 inches high. If the magnolia tree grows 10 inches a year, how many years will pass before the tree is 16 feet high?

   The unknown quantity in this problem is the number of years it will take the tree to grow to be 16 feet tall. Let y equal the number of years. Then $10y$ is the number of inches the tree will grow in y years.

   Write the heights in inches. now: 7 feet 3 inches = 87 inches
   in y years: 16 feet = 192 inches

   Write the problem in words. 87 inches plus 10 times some number of years equals 192 inches.

   Write an equation. $87 + 10y = 192$

MODEL ACT PROBLEM

A newspaper reporter can write w words per hour. Her colleague can write $w - 15$ words per hour. If they work together on a story, which equation below shows the amount of time, t, it would take them to write 1,500 words?

A. $t = w^2 - 15w - 1{,}500$

B. $t = \frac{1{,}500}{2w - 15}$

C. $t = 1{,}500(2w - 15)$

D. $t = 1{,}515 - 2w$

E. $t = 100$

SOLUTION

Find the number of words the reporters can write in total in one hour.

$w + (w - 15) = 2w - 15$

Divide the total number of words by the number of words written per hour to find the total number of hours.

$\text{total hours} = \frac{1{,}500}{2w - 15}$

The correct answer is B.

Practice

Write an expression for each of these statements.

1. The difference of the money in a bank account at the beginning of the year and the amount withdrawn during the year.
2. The average of Scott's grades on 5 quizzes.
3. Laura's pay for one week if she worked 8 hours at her regular rate and 3 hours of overtime at 1.5 times her regular rate.
4. The amount of money in a savings account at the end of a year if P dollars were deposited at the beginning of the year and I% interest was earned on the account.

Write an equation that can be used to solve each problem.

5. The balance of Brynah's account at the Credit Union was \$3,155. After she made a withdrawal to pay for car repairs her new balance was \$2,855. How much did Brynah withdraw from her account?
6. When Tony turned 9 years old he received an allowance of \$4 each week. His parents increased his allowance by \$0.75 a week each birthday. What was Tony's allowance when he turned 12?
7. A chemistry researcher has a container filled with 100 ml of hydrochloric acid (HCl). If the researcher does a series of experiments that each use 4 ml of HCl, how much of the acid is left?
8. To design a house, an architect charges a certain fee, f, per square foot of the house. If he gives the Martins a 25% discount off that fee, how much will they pay for the design of an 8,000-square-foot house?
9. Write an equation that could be used to find Marvin's age if 3 more than Kim's age is equal to 29 less than Marvin's age.
10. The tax on a restaurant meal is 8%. If Joe paid a 15% tip on the original (pre-tax) meal cost, write an equation to show how much Joe paid in total for a meal whose original cost was \$20.

(Answers on page 345)

ACT-TYPE PROBLEMS

1. A computer virus creates a total of 50 copies of itself on each computer it infects. If n computers at a school are infected with the virus, which equation shows V, the number of copies of the virus at that school?

   A. $V = n^{50}$
   B. $V = 50^n$
   C. $V = 50 + n$
   D. $V = 50n$
   E. $V = \frac{50}{n}$

2. A company starts with \$500,000 in a savings account. Every 2 years the account grows by \$250,000 in regular yearly amounts. Which of the following expressions can be used to represent the amount in dollars in the account after x years?

   F. $500{,}000 + \frac{250{,}000}{x}$
   G. $500{,}000 + \left(\frac{250{,}000}{2}\right)x$
   H. $250{,}000 + 250{,}000x$
   J. $500{,}000 + \left(\frac{500{,}000}{2}\right)x$
   K. $250{,}000 + \left(\frac{250{,}000}{2}\right)x$

3. A bag of 30 chocolate bars is shared equally among 12 children. Which equation could be used to find the number of chocolate bars each child receives?

   A. $c = 12 \times 30$
   B. $c = 30 - 12$
   C. $c = 12 \div 30$
   D. $c = 30 \div 12$
   E. $c = 12 + 30$

4. A bicycle store pays sales representatives a base salary of \$5 per hour worked. In addition, employees receive a commission of 8% of the value of each bicycle sold. Which expression shows Joan's pay for the week if she worked h hours and sold \$$v$ worth of bicycles?

   F. $5h + 0.08v$
   G. $5v + 0.08h$
   H. $5h + 1.08v$
   J. $5.08(h + v)$
   K. $5.08hv$

5. Rachel has an average of 82 on the last four math tests. If she has one more test to take before the end of the semester, which equation could be used to find the score she needs to raise her average to an 85?

   A. $\frac{82 + T}{5} = 85$
   B. $\frac{82 + T}{2} = 85$
   C. $T = \frac{82 + 85}{2}$
   D. $\frac{4(82) + T}{2} = 85$
   E. $\frac{4(82) + T}{5} = 85$

(Answers on page 345)

Solving Linear Equations

Algebraic symbols are either constants or variables.

A **constant** is one value.

Examples: $2, \frac{1}{2}, 0.08, \pi, \sqrt{5}$

A **variable** may have one or more values.

Examples: x, y, a, t

When solving equations, first add or subtract, then multiply or divide.

Solving an equation means finding the value of the variable that makes the equation true. Solving an equation is a mechanical process. The idea is to get the variable on one side of the equal sign and the value of the variable (a number) on the other side of the equal sign. You can add or subtract any number on each side of the equal sign, or you can multiply or divide each side of the equation by the same *nonzero* number. Substitute your solution back into the equation to check that the solution is correct.

CALCULATOR TIP

Some graphing calculators can solve linear equations. If the solution is simple, however, it may be easier to use paper and pencil.

Using Addition or Subtraction to Solve Equations

EXAMPLES

1. Solve. $x + 27 = 48$

$$x + 27 = 48$$

Subtract 27. $\underline{-27 \quad -27}$

$$x = 21$$

Check: substitute 21 for x.

$$21 + 27 \stackrel{?}{=} 48$$

$$48 = 48 \checkmark$$

2. Solve. $-47 + x = -63$

$$-47 + x = -63$$

Add 47. $\underline{+47 \quad +47}$

$$x = -16$$

Check: substitute -16 for x.

$$-47 + (-16) \stackrel{?}{=} -63$$

$$-63 = -63 \checkmark$$

Using Multiplication or Division to Solve Equations

EXAMPLES

1. Solve. $\frac{y}{4} = 15$

$$\frac{y}{4} = 15$$

Multiply by 4. $4\left(\frac{y}{4}\right) = 4(15)$

$$y = 60$$

Check: substitute 60 for y.

$$\frac{60}{4} \stackrel{?}{=} 15$$

$$15 = 15 \checkmark$$

2. Solve. $4k = 61$

$$4k = 61$$

Divide by 4. $\frac{4k}{4} = \frac{61}{4}$

$$k = 15\frac{1}{4}$$

Check: substitute $15\frac{1}{4}$ for k.

$$4\left(15\frac{1}{4}\right) \stackrel{?}{=} 61$$

$$61 = 61 \checkmark$$

Solving Equations in Two or More Steps

You may have to use several steps to solve an equation. Always add or subtract before you multiply or divide.

EXAMPLES

1. Solve. $5t - 8.5 = 54$

$$5t - 8.5 = 54$$

Add 8.5. $\underline{+8.5 \quad +8.5}$

$$5t = 62.5$$

Divide by 5. $\frac{5t}{5} = \frac{62.5}{5}$

$$t = 12.5$$

Check: substitute 12.5 for t.

$$5(12.5) - 8.5 \stackrel{?}{=} 54$$

$$62.5 - 8.5 \stackrel{?}{=} 54$$

$$54 = 54 \checkmark$$

2. Solve. $\frac{k}{8} + 3.4 = -28$

$$\frac{k}{8} + 3.4 = -28$$

Subtract 3.4. $\underline{-3.4 \quad -3.4}$

$$\frac{k}{8} = -31.4$$

Multiply by 8. $8\left(\frac{k}{8}\right) = 8(-31.4)$

$$k = -251.2$$

Check: substitute -251.2 for k.

$$\frac{-251.2}{8} + 3.4 \stackrel{?}{=} -28$$

$$-31.4 + 3.4 \stackrel{?}{=} -28$$

$$-28 = -28 \checkmark$$

3. Solve. $3x - 4 = 5 + x$

$$3x - 4 = 5 + x$$

Subtract x. $\underline{-x \quad -x}$

$$2x - 4 = 5$$

Add 4. $\underline{+4 \quad +4}$

$$2x = 9$$

Divide by 2. $\frac{2x}{2} = \frac{9}{2}$

$$x = 4.5$$

Check: substitute 4.5 for x.

$$3(4.5) - 4 \stackrel{?}{=} 5 + 4.5$$

$$13.5 - 4 \stackrel{?}{=} 9.5$$

$$9.5 = 9.5 \checkmark$$

Solving Literal Equations

A **literal equation** is an equation with two or more different variables. For instance, $ax + y = c$ is a literal equation that can be solved for a, x, y, or c.

EXAMPLE

Solve $ax + y = c$ for x.

$$ax + y = c$$

Subtract y. $\underline{-y \quad -y}$

$$ax = c - y$$

Divide by a. $\frac{ax}{a} = \frac{c - y}{a}$

$$x = \frac{c - y}{a}$$

Solving Fractional Equations

Follow these steps to solve fractional equations.

1. Isolate the variable on one side of the equal sign.
2. Multiply by the inverse of the coefficient to solve the problem.

EXAMPLE

Solve. $2\frac{2}{3}n + 31 = -21$

$$2\frac{2}{3}n + 31 = -21$$

Subtract 31. $\underline{-31 \quad -31}$

$$2\frac{2}{3}n = -52$$

Write $2\frac{2}{3}$ as a fraction. $\frac{8}{3}n = -52$

Multiply by the reciprocal, $\frac{3}{8}$. $\frac{3}{8}\left(\frac{8}{3}n\right) = \frac{3}{8}(-52)$

$$n = -19\frac{1}{2}$$

MODEL ACT PROBLEMS

1. If $3s + 2 = 8$, what is the value of s?

A. 2
B. 3
C. 4
D. 5
E. 6

SOLUTION

Write the equation and solve.

	$3s + 2$	$= 8$
Subtract 2.	$3s$	$= 6$
Divide by 3.	s	$= 2$

The correct answer is A.

2. Robert's age multiplied by 5, plus 6, equals 41. How old is Robert?

F. 4 years old
G. 5 years old
H. 6 years old
J. 7 years old
K. 8 years old

SOLUTION

Write an equation and solve for Robert's age.

Let r = Robert's age.

	$5r + 6$	$= 41$
Subtract 6.	$5r$	$= 35$
Divide by 5.	r	$= 7$ years old

The correct answer is J.

Practice

Solve each equation. Check each answer.

1. $w + 9 = 25$
2. $-18 = x + 3$
3. $3y = 15$
4. $8z = -28$
5. $30y + 7 = 13$
6. $3y + 8 = -5y + 88$
7. $3v - 9 = -v + 27$
8. $10 - 3t = -5t - 8$
9. $-1.8 = 3m + 3$
10. $2n - 6 = 10 - 14n$
11. $-19 - 6w = -37$
12. $0.5s - 3 = -14$
13. $-7d + 3 = -18$
14. $4g + 6 = 8$
15. $-2k - 3 = 2$
16. Solve $2x + y = c$ for x.
17. Solve $m - 3k = p$ for k.
18. Solve $-x + y = z$ for x.
19. Solve $3k - 2y = -r$ for k.

Solve each equation.

20. $\frac{2}{3}x + 12 = 28$
21. $1\frac{1}{7}y - 9 = 27$
22. $\frac{1}{8}k - 11 = 21$
23. $1\frac{3}{8}a + \frac{1}{4} = \frac{7}{8}$
24. $\frac{2}{5}b + \frac{3}{7} = \frac{11}{5}$
25. $\frac{3}{7}t + \frac{4}{5} = \frac{6}{7}$
26. Jim's weekly pay is half Carl's weekly pay. If Carl is paid $225 a week, how much is Jim paid a week?
27. Henry hit 60 home runs. Ken hit a third fewer home runs than Henry, plus 30. How many home runs did Ken hit?

28. What is the sum of a and b, if $6a + 2 = 14$, and b is 5 more than a?
29. What is three times the value of x if $-4x - 5 = -15$?
30. Julia runs 5 miles a day. Phil runs twice as far each day as Julia, minus 5 miles. How many miles does Phil run each day?

(Answers on page 345)

ACT-TYPE PROBLEMS

1. Solve. $5y - 3y + 13 = 26$

A. −3
B. 5
C. 5.5
D. 6.5
E. 13

2. Given the two linear equations $4r - 5 = 17$ and $12 - 2s = 2$, what is the product of r and s?

F. 5
G. 5.5
H. 10.5
J. 25
K. 27.5

3. Emily is half Mike's age, plus 7. If Mike is 28 years old, how old is Emily?

A. 17 years old
B. 19 years old
C. 21 years old
D. 23 years old
E. 25 years old

4. Solve for h. $\frac{2}{3}h - 5 = h + 7$

F. −36
G. 36
H. −24
J. 24
K. 20

5. The equation $\$2,225c - \$2,000 = P$ shows the profit (P) a computer store makes selling (c) computers. How many computers must the store sell to make a profit of $49,175?

A. 21 computers
B. 22 computers
C. 23 computers
D. 24 computers
E. 25 computers

(Answers on page 345)

7. If you roll a pair of fair six-sided dice, what is the probability that their sum will be 7?

A. $\frac{1}{36}$
B. $\frac{1}{18}$
C. $\frac{1}{12}$
D. $\frac{1}{9}$
E. $\frac{1}{6}$

8. Given the numbers $\frac{1}{2}, \frac{2}{8}, \frac{3}{5}, \frac{7}{20}, \frac{11}{40}$, which of the following lists the numbers from smallest to largest?

F. $\frac{3}{5}, \frac{2}{8}, \frac{11}{40}, \frac{7}{20}, \frac{1}{2}$
G. $\frac{2}{8}, \frac{11}{40}, \frac{7}{20}, \frac{1}{2}, \frac{3}{5}$
H. $\frac{1}{2}, \frac{3}{5}, \frac{2}{8}, \frac{7}{20}, \frac{11}{40}$
J. $\frac{11}{40}, \frac{7}{20}, \frac{2}{8}, \frac{3}{5}, \frac{1}{2}$
K. $\frac{7}{20}, \frac{11}{40}, \frac{1}{2}, \frac{2}{8}, \frac{3}{5}$

9. Ed can lift two 50-pound weights and two 25-pound weights on a lifting bar that already weighs 45 pounds. Which of the following choices displays in scientific notation the total weight Ed can lift?

A. 1.2×10^2 pounds
B. 1.5×10^2 pounds
C. 1.2×10^{-2} pounds
D. 1.95×10^2 pounds
E. 1.95×10^{-2} pounds

10. A catering service needs to have 4 tables for every 28 people attending a party. If 224 people are attending the party, how may tables will the catering service need?

F. 8
G. 16
H. 24
J. 32
K. 40

11. What is the value of the following expression rounded to the nearest 100th?

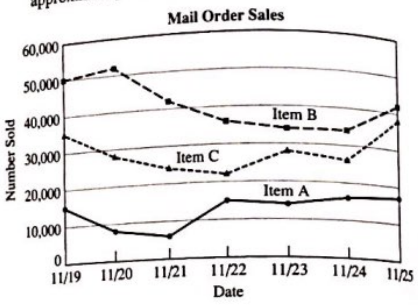
$$\frac{3 \times \frac{1}{4} + 5 + 0.2 - 3.7 + 2 - \frac{1}{3}}{8 - 2.1 + \frac{2}{5} \times 8}$$

A. 0.4
B. 0.43
C. 0.431
D. 0.44
E. 0.5

12. There are three tollbooths open on a toll road. Four cars are headed for the exact-change tollbooth. In how many ways can the cars line up to go through the one exact-change toll?

F. 1
G. 4
H. 12
J. 24
K. 36

13. A large mail order business sells hundreds of thousands of items. The line graph shows the sales levels of three of these items (Item A, Item B, and Item C) from November 19 to November 25. On which date was the sales level of Item A approximately half the sales level of Item C?



A. November 19
B. November 20
C. November 22
D. November 23
E. November 25

14. The stem-and-leaf plot below shows attendance at the home basketball games this season. What is the difference between the mode and the median attendance at these games?

11	0, 0, 2, 3
13	4, 5, 8, 9
14	5, 7, 9
15	2, 4, 6
16	0, 1, 3

F. 7
G. 29
H. 35
J. 110
K. 145

Chapter 8

Elementary Algebra

- Ten ACT questions have to do with elementary algebra.
- Easier elementary algebra questions may be about a single skill or concept or may test a combination of pre-algebra and elementary algebra skills.
- More difficult questions will often test a combination of elementary algebra skills and concepts.
- This elementary algebra review covers all the material you need to answer ACT questions.
- Use a calculator for the ACT-Type Problems. Do not use a calculator for the Practice exercises.

Evaluating Formulas and Expressions

Formulas and **expressions** are statements about numbers. To **evaluate** a formula or an expression, substitute values for the variables and then compute. Remember to use the correct order of operations.

Expressions do not have equal signs. You evaluate formulas and expressions, you don't solve them.

CALCULATOR TIP

Some graphing calculators can evaluate expressions. However, it may be easier to evaluate the expression using paper and pencil.

EXAMPLES

1. The formula for the area of a square is $A = s^2$, where s is the length of the side of the square.

   Evaluate $A = s^2$ for $s = 4$ centimeters.

   $A = 4^2 = 16 \text{ cm}^2$

2. The formula for the area of a circle is $A = \pi r^2$, where r is the length of the radius.

   Evaluate $A = \pi r^2$ for $r = 3$ inches. (Use 3.14 for π.)

   $A = (3.14)(3^2) = 3.14 \times 9 = 28.26 \text{ in.}^2$

Use square units for area.

NUMBERS
1, 2, 3, 4 44–48
48–51

3. The formula for the area of a trapezoid is $A = \frac{h}{2}(b_1 + b_2)$, where b_1 and b_2 are the lengths of the two parallel bases and h is the height.

Evaluate $A = \frac{h}{2}(b_1 + b_2)$ for $h = 3.5$ meters, $b_1 = 4.4$ meters, and $b_2 =$ 5.8 meters.

$A = \frac{3.5}{2}(4.4 + 5.8) = \frac{3.5}{2}(10.2) = 1.75(10.2) = 17.85 \text{ m}^2$

4. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

Evaluate $V = \frac{4}{3}\pi r^3$ for $r = 6$ centimeters. (Use 3.14 for π.)

$V = \left(\frac{4}{3}\right)(3.14)(6^3) = 904.32 \text{ m}^3$

Use cubic units for volume.

5. The formula for simple interest is $I = PRT$.

Simple interest is the amount paid or earned in interest.

P = Principal, the amount borrowed or deposited.

R = Rate, the interest rate.

T = Time, the length of the loan or deposit in years. (For example, for 6 months, $T = \frac{1}{2}$. For 3 months, $T = \frac{1}{4}$.)

Evaluate $I = PRT$ for $P = \$2{,}800$, $R = 9.8\%$, and $T = 30$ months.

Write the percent as a decimal. $9.8\% = 0.098$

Write T in years. $30 \text{ months} = \frac{30}{12} \text{ years} = 2.5 \text{ years}$

$I = (\$2{,}800)(0.098)(2.5) = \$686$

The interest is $686.

6. If $y = 3$, $s = 2.8$, and $k = \frac{1}{4}$, what is the value of $3y - 4s + \frac{y}{k}$?

Substitute the numerical value for each variable and then compute.

$3y - 4s + \frac{y}{k}$

$= 3(3) - 4(2.8) + \frac{3}{\frac{1}{4}}$ Substitute. Multiply or divide from left to right.

$\left(\text{Remember: } \frac{3}{\frac{1}{4}} = \frac{3}{1} \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12.\right)$

$= 9 - 11.2 + 12$ Add or subtract from left to right.

$= 9.8$

The answer is 9.8.

MODEL ACT PROBLEM

What is the radius of a circle if the area is 49π in.²?

A. 24.5 in.
B. 20 in.
C. 14 in.
D. 7 in.
E. 3.5 in.

SOLUTION

Use the area formula.	A	=	πr^2
Substitute 49π for A.	49π	=	πr^2
Divide by π.	$\frac{49\pi}{\pi}$	=	$\frac{\pi r^2}{\pi}$
Evaluate.	$\sqrt{49}$	=	$\sqrt{r^2}$
	7 in.	=	r

The correct answer is D.

Practice

Use these formulas to help you complete the practice exercises that follow.

Geometric Formulas

Triangle	Area $= \frac{1}{2}bh$	Circle	Area $= \pi r^2$
Square	Area $= s^2$		Circumference $= 2\pi r$ or πd
Rectangle	Area $= lw$	Cube	Volume $= s^3$
Parallelogram	Area $= bh$	Rectangular Prism	Volume $= lwh$
Trapezoid	Area $= \frac{1}{2}h(b_1 + b_2)$	Sphere	Volume $= \frac{4}{3}\pi r^3$

For exercises 1–12, all measurements are in centimeters. Use 3.14 for π.

Find the area of each figure.

1. Triangle: $b = 3$, $h = 8$
2. Square: $s = 0.7$
3. Rectangle: $l = 1.5$, $w = 1.2$
4. Parallelogram: $b = 2.7$, $h = 1.3$
5. Trapezoid: $b_1 = 4$, $b_2 = 0.5$, $h = 1.3$
6. Triangle: $b = \frac{1}{2}$, $h = \frac{1}{4}$

Find the circumference and the area of each circle.

7. Circle: $r = 2$
8. Circle: $d = 6$
9. Circle: $d = 1.8$

Find the volume of each solid.

10. Cube: $s = 0.9$
11. Rectangular prism: $l = 4$, $w = 6$, $h = 1.5$
12. Sphere: $r = 3$

1, 2, 3, 4

Use the formula $d = rt$, where d = distance, r = rate, and t = time, to find the missing value.

13. d = ____?____
r = 60 mph
t = 2 hours

14. d = 270 feet
r = ____?____
t = 90 seconds

15. d = 420 miles
r = 70 mph
t = ____?____

Use a formula to solve each problem.

16. At an annual rate of 11.25%, what is the simple interest earned in 24 months on an account that has a principal amount of $7,500?

17. What is the circumference of a circle with a diameter of 13 centimeters?

18. What is the average speed of a car that travels 290 miles in 5 hours?

19. The height of a trapezoid is 5 inches. One base is twice the height and the other base is half the height. What is the area of the trapezoid?

20. What is the volume of a sphere with the same radius as a circle with a diameter of 30 millimeters?

21. Belinda has $1,025 in a savings account that pays 4% simple interest. How much will she have in her account after two years if she makes no deposits or withdrawals?

22. Ricardo borrowed $8,500 to buy a car. How much interest will he pay if the loan is for 4 years at 8% simple interest?

(Answers on page 348)

ACT-TYPE PROBLEMS

1. What is the volume of a cube if the area of one face of the cube is 36 m²?

A. 12 m^3
B. 18 m^3
C. 108 m^3
D. 206 m^3
E. 216 m^3

2. Add the sum of the sides of a square with an area of 30.25 in.² to the sum of the edges of a cube with a volume of 343 in.³.

F. 5.5 in.
G. 7 in.
H. 12.5 in.
J. 64 in.
K. 106 in.

3. What is the area of a circle with a circumference of 24π cm?

A. 12π cm²
B. 24π cm²
C. 120π cm²
D. 144π cm²
E. 240π cm²

4. The area of a rectangle is 30 in.² and the length of the rectangle is 6 in. What is the sum of the length and the width of the rectangle?

F. 11 in.
G. 9 in.
H. 7 in.
J. 5 in.
K. 3 in.

5. John and Eric drove the same distance in separate cars. It took John 6 hours to complete the trip, traveling at an average speed of 65 mph. How much longer would the trip take Eric, if Eric traveled at an average speed of 60 mph?

A. $\frac{1}{4}$ hour
B. $\frac{1}{2}$ hour
C. $\frac{3}{4}$ hour
D. 1 hour
E. 2 hours

(Answers on page 348)

Exponents and Radicals

Exponents

- The **base** shows the factor.
- The **exponent** shows how many times the factor is repeated.

Exponent ↓

Base → $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

The factor is 2.

Use the following rules for exponents.

Rule	*Example*
$x^0 = 1\ (x \neq 0)$	$16^0 = 1$
$x^1 = x$	$16^1 = 16$
$(x^n)(x^m) = x^{n+m}$	$7^8 \times 7^5 = 7^{13}$
$\frac{x^n}{x^m} = x^{n-m}\ (x \neq 0)$	$\frac{7^8}{7^5} = 7^3$
$(x^m)^n = x^{mn}$	$(5^2)^3 = 5^6$
$x^{-m} = \frac{1}{x^m}\ (x \neq 0)$	$5^{-4} = \frac{1}{5^4}$

Radicals

Write radicals with the radical sign or as a fractional power.

Index ↓

$8\sqrt[3]{7} = 8 \times 7^{\frac{1}{3}}$

↗ Coefficient ↖ Radicand

This radical shows *8 times the cube root of 7*.

Simplifying Radicals

Simplify all radicals so that:

- There are no fractions in the radicand.
- All nth powers of whole numbers are removed from the radicand.
- There are no radicals in the denominator.
- The index is as low as possible.

$\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$ and $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Removing the radical sign from the denominator is called **rationalizing the denominator.**

CALCULATOR TIP

Use the exponent key to find the value of a radical. For example, $\sqrt[5]{a} = a^{\frac{1}{5}}$.

EXAMPLES

1. Simplify. $\sqrt{75}$
   Look for a perfect square factor. $\sqrt{75} = \sqrt{25 \cdot 3}$
   Extract the square factor. $= \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

2. Simplify. $\sqrt{\frac{7}{16}}$
   Write as separate radicals. $\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}}$
   Find the square root of the denominator. $= \frac{\sqrt{7}}{4}$

3. Simplify. $\sqrt{\frac{3}{7}}$
   Write as separate radicals. $\sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}}$
   Multiply the numerator and denominator by $\sqrt{7}$ to remove the radical from the denominator. $= \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$

MODEL ACT PROBLEM

1. What is the sum of the factors of $3^4 \cdot 4^3 \cdot 5^2$?

   A. 9
   B. 12
   C. 24
   D. 34
   E. 60

SOLUTION

Write the factors.

$3^4 \cdot 4^3 \cdot 5^2 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 4 \cdot 4 \cdot 5 \cdot 5$

Add the factors.

$3 + 3 + 3 + 3 + 4 + 4 + 4 + 5 + 5 = 34$

The correct answer is D.

2. Simplify $\frac{3^2}{\sqrt{80}}$.

   F. $\frac{\sqrt{80}}{3^{-2}}$
   G. $\frac{4}{9}\sqrt{5}$
   H. $\frac{5}{4}\sqrt{5}$
   J. $\frac{9}{20}\sqrt{5}$
   K. $\frac{9}{4}\sqrt{5}$

SOLUTION

Simplify the numerator. Factor and simplify the denominator.

$\frac{3^2}{\sqrt{80}} = \frac{9}{\sqrt{16}\sqrt{5}} = \frac{9}{4\sqrt{5}}$

Multiply numerator and denominator by $\sqrt{5}$ to remove the radical from the denominator.

$\frac{9\sqrt{5}}{4\sqrt{5}\sqrt{5}} = \frac{9\sqrt{5}}{4 \cdot 5} = \frac{9}{20}\sqrt{5}$

The correct answer is J.

Practice

Compute.

1. $7^8 \times 7^3$
2. $9^{-4} \times 9^7$
3. $2^3 \times (3^2)^2$
4. $3^5 \times 3^{-5}$
5. $3^3 \times 9^2$
6. $5^2 \div 5^5$
7. $4^2 \div 2^4$
8. $12^5 \div 12^{-5}$
9. $4^{\frac{1}{2}} \div 4^1$
10. $6^7 \div 6^2$

Simplify.

11. $\sqrt{25}$
12. $\sqrt{864}$
13. $\sqrt[3]{56}$
14. $\sqrt[3]{576}$
15. $\sqrt{162}$
16. $\frac{7}{\sqrt{11}}$
17. $\frac{6}{\sqrt{18}}$
18. $\frac{19}{\sqrt{19}}$
19. $\frac{13}{\sqrt[3]{296}}$
20. $\frac{13}{\sqrt{8}}$

(Answers on page 349)

ACT-TYPE PROBLEMS

1. $13^{-2} \times 13^2 = ?$

   A. 2
   B. 1
   C. 0
   D. −1
   E. −2

2. $\frac{7}{\sqrt{45}} = ?$

   F. $\frac{7}{3}$
   G. $\frac{7}{9}$
   H. $\frac{3}{7}$
   J. $\frac{7\sqrt{3}}{9}$
   K. $\frac{7\sqrt{5}}{15}$

3. $5^4 \div 25^2 = ?$

   A. 20
   B. 15
   C. 10
   D. 5
   E. 1

4. Which of the following choices is equal to $\frac{4^7}{\sqrt{16}}$?

   F. 4,094
   G. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 6 \times 6$
   H. 2^{12}
   J. 8^3
   K. $2^6 \times 8^4$

5. All of the following choices are the reciprocal of $\frac{\sqrt{72}}{6^5}$ EXCEPT

   A. $(6^2)^2 \times \frac{\sqrt{2}}{2}$
   B. $6 \times 6^3 \times \frac{\sqrt{2}}{2}$
   C. $6^2 \times 6^2 \times \frac{\sqrt{2}}{2}$
   D. $6^8 \div 6^4 \times \frac{\sqrt{2}}{2}$
   E. $6^{12} \div 6^3 \times \frac{\sqrt{2}}{2}$

(Answers on page 349)

Operations With Radicals

You can add, subtract, multiply, and divide radicals.

> You can add or subtract only when the radicals are the same.

Addition and Subtraction

To add or subtract radicals:

- Rewrite so the radicals are the same.
- Add or subtract the coefficients.

EXAMPLES

1. Add. $5\sqrt{7} + 2\sqrt{112}$
   - Rewrite $2\sqrt{112}$. $\quad 2\sqrt{112} = 2\sqrt{16 \times 7} = 2 \times 4\sqrt{7} = 8\sqrt{7}$
   - Add the coefficients. $\quad 5\sqrt{7} + 8\sqrt{7} = 13\sqrt{7}$
   - So, $5\sqrt{7} + 2\sqrt{112} = 13\sqrt{7}$.

2. Add. $3\sqrt[3]{162} + 8\sqrt[3]{6}$
   - Rewrite $3\sqrt[3]{162}$. $\quad 3\sqrt[3]{162} = 3\sqrt[3]{27 \times 6} = 3 \times 3\sqrt[3]{6} = 9\sqrt[3]{6}$
   - Add the coefficients. $\quad 9\sqrt[3]{6} + 8\sqrt[3]{6} = 17\sqrt[3]{6}$
   - So, $3\sqrt[3]{162} + 8\sqrt[3]{6} = 17\sqrt[3]{6}$.

3. Subtract. $3\sqrt{48} - \sqrt{27}$
   - Rewrite $3\sqrt{48}$. $\quad 3\sqrt{48} = 3\sqrt{16 \times 3} = 12\sqrt{3}$
   - Rewrite $\sqrt{27}$. $\quad \sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$
   - Subtract the coefficients. $\quad 12\sqrt{3} - 3\sqrt{3} = 9\sqrt{3}$
   - So, $3\sqrt{48} - \sqrt{27} = 9\sqrt{3}$.

CALCULATOR TIP

If the answers to a question are in radical form, check to see if your calculator represents answers in radical form. Graphing calculators usually don't have this feature.

Multiplication

To multiply radicals:

- Write the factors under one radical sign.
- Multiply.
- Simplify if possible.

> Radicals do NOT need to be the same to multiply.

EXAMPLE

Multiply. $\sqrt{10} \times \sqrt{15}$

- Write factors under one radical sign. $\quad \sqrt{10} \times \sqrt{15} = \sqrt{10 \times 15}$
- Multiply. $\quad = \sqrt{150}$
- Simplify if possible. $\quad = \sqrt{25 \times 6} = 5\sqrt{6}$
- So, $\sqrt{10} \times \sqrt{15} = 5\sqrt{6}$.

Division

To divide radicals:

- Write as a fraction under one radical sign.
- Factor, cancel, and simplify.
- The denominator of the answer must be a whole number.

EXAMPLES

Divide. $\dfrac{\sqrt{10}}{\sqrt{15}}$

- Rewrite under one radical sign. $\quad \dfrac{\sqrt{10}}{\sqrt{15}} = \sqrt{\dfrac{10}{15}}$
- Factor numerator and denominator. Cancel. $\quad = \sqrt{\dfrac{2 \times \cancel{5}}{3 \times \cancel{5}}} = \sqrt{\dfrac{2}{3}}$
- The denominator is not a whole number. Multiply numerator and denominator by the denominator. $\quad = \sqrt{\dfrac{2 \times 3}{3 \times 3}} = \sqrt{\dfrac{6}{9}}$
- Simplify. $\quad = \dfrac{\sqrt{6}}{3}$
- So, $\dfrac{\sqrt{10}}{\sqrt{15}} = \dfrac{\sqrt{6}}{3}$.

MODEL ACT PROBLEMS

1. $3\sqrt{48} + 11\sqrt{75} = ?$
   - **A.** $14\sqrt{75}$
   - **B.** $33\sqrt{48}$
   - **C.** $67\sqrt{3}$
   - **D.** $72\sqrt{5}$
   - **E.** $81\sqrt{2}$

SOLUTION

Rewrite with radicals that are the same. Then add the coefficients.

$3\sqrt{48} + 11\sqrt{75} = 3\sqrt{16 \times 3} + 11\sqrt{25 \times 3}$

$= 3 \times 4\sqrt{3} + 11 \times 5\sqrt{3} = 12\sqrt{3} + 55\sqrt{3} = 67\sqrt{3}$

The correct answer is C.

2. $\sqrt{14} \div \sqrt{6} = ?$
   - **F.** $\frac{1}{7}$
   - **G.** $\frac{2}{3}$
   - **H.** $\frac{21}{\sqrt{3}}$
   - **J.** $\frac{3}{\sqrt{21}}$
   - **K.** $\frac{\sqrt{21}}{3}$

SOLUTION

Rewrite the factors under one radical sign. Then simplify.

$\dfrac{\sqrt{14}}{\sqrt{6}} = \sqrt{\dfrac{14}{6}} = \sqrt{\dfrac{2 \cdot 7}{2 \cdot 3}} = \sqrt{\dfrac{7}{3}} = \sqrt{\dfrac{7 \cdot 3}{3 \cdot 3}} = \sqrt{\dfrac{21}{9}} = \dfrac{\sqrt{21}}{3}$

The correct answer is K.

Practice

Add.

1. $5\sqrt{6} + 3\sqrt{216}$
2. $6\sqrt[3]{7} + 8\sqrt[3]{875}$
3. $9\sqrt{48} + 11\sqrt{75}$

Subtract.

4. $4\sqrt{50} - 12\sqrt{72}$
5. $9\sqrt{432} - 4\sqrt{1{,}323}$
6. $11\sqrt[3]{96} - 4\sqrt[3]{324}$

Multiply.

7. $\sqrt{8} \times \sqrt{7}$
8. $\sqrt{3} \times \sqrt{39}$
9. $\sqrt{8} \times \sqrt{30}$

Divide.

10. $\frac{\sqrt{3}}{\sqrt{39}}$
11. $\frac{\sqrt{72}}{\sqrt{9}}$
12. $\frac{\sqrt{15}}{\sqrt{135}}$

Simplify.

13. $\sqrt{3} \times \sqrt{6} + \sqrt{72}$
14. $\sqrt{13} - \sqrt{52} + \sqrt{208}$
15. $-\sqrt{3} + \sqrt{5} \times \sqrt{15}$
16. $\frac{\sqrt{13}}{\sqrt{117}} - \frac{\sqrt{1}}{\sqrt{16}}$
17. $\frac{\sqrt{12} + \sqrt{27}}{\sqrt{6}}$
18. $\sqrt{6} \times \sqrt{6} - 6 + \sqrt{16}$
19. $\sqrt{32} - \sqrt{8} - 2\sqrt{2}$
20. $\sqrt{3} \times \sqrt{5} - \sqrt{375}$

(Answers on page 349)

ACT-TYPE PROBLEMS

1. $\sqrt{22} \times \sqrt{14} = ?$
   - A. $-4\sqrt{77}$
   - B. $-2\sqrt{77}$
   - C. $2\sqrt{77}$
   - D. $4\sqrt{77}$
   - E. $8\sqrt{77}$

2. $\sqrt{54} - \sqrt{96} = ?$
   - F. $-2\sqrt{3}$
   - G. $-\sqrt{6}$
   - H. $2\sqrt{3}$
   - J. $\sqrt{6}$
   - K. $7\sqrt{6}$

3. $3\sqrt{125} + 6\sqrt{80} = ?$
   - A. -45
   - B. $-29\sqrt{5}$
   - C. $-9\sqrt{5}$
   - D. $39\sqrt{5}$
   - E. 145

4. $\sqrt{8}(\sqrt{13} - \sqrt{117}) = ?$
   - F. $-4\sqrt{26}$
   - G. $4\sqrt{2}$
   - H. $2\sqrt{13}$
   - J. $2\sqrt{26}$
   - K. $4\sqrt{26}$

5. What is the product of the numerator and denominator in the simplified form of $\frac{\sqrt{45}}{\sqrt{10}}$?
   - A. $\sqrt{6}$
   - B. $\frac{3}{2}\sqrt{6}$
   - C. $3\sqrt{2}$
   - D. $6\sqrt{2}$
   - E. 18

(Answers on page 349)

Polynomials

Types of Polynomials

Polynomial is just another name for an expression. Recall that expressions do not contain equal signs. There are different types of polynomials.

A **monomial** can be a constant, a variable, or the product or quotient of constants and variables.

Notice that there are no addition or subtraction signs in monomials.

Examples: $5, z, 4x, y^5, 345, xy^2z^4w^5, \frac{3}{8}, -8.6$

A **binomial** is the sum or difference of two monomials. Each monomial is called a term.

Examples: $3x - 2, 5x^3 + 7, -8 + 62x^4y^3, 7z^5y^2 - 8x^3z^2$

A **trinomial** is the sum or difference of three monomials. Each monomial is called a term.

Examples: $9x^5 + 45y^3 - 6y, z^5 - x^3 - y^8, 7x^2 + 5x + 34$

Similar Terms

Terms consist of constants—called coefficients—and variables. In the term $5x$, the coefficient is 5 and the variable is x.

Similar terms have exactly the same variable part. The order of the variables is not important.

Examples: $5xy$ is similar to $15xy$ and $34yx$.
$6x^2y$ is similar to $17yx^2$ and x^2y.
$6x^2y$ is NOT similar to $5xy^2$ or to $7y^2x$.

To combine similar terms, add or subtract the coefficient and keep the variable part.

EXAMPLE

Combine similar terms. $3x + 2xy + 7y - 5x + 3x^2 - 2y$

Group similar terms.	$(3x - 5x) + (7y - 2y) + 2xy + 3x^2$
Combine similar terms.	$-2x \quad + \quad 5y \quad + 2xy + 3x^2$

Always combine similar terms.

Operations on Polynomials

Adding Polynomials

To add polynomials, combine similar terms.

CALCULATOR TIP

The calculators allowed on the ACT add, subtract, multiply, or divide polynomials. You will have to use paper and pencil.

EXAMPLE

Add. $(3x^4 + 5x^2y - x^2 + 8) + (3x^5 - 6x^2y + 7y^2x - 16)$

Remove the parentheses.	$3x^4 + 5x^2y - x^2 + 8 + 3x^5 - 6x^2y + 7y^2x - 16$
Group similar terms.	$3x^5 + 3x^4 + (5x^2y - 6x^2y) - x^2 - 7y^2x + (8 - 16)$
Combine similar terms.	$3x^5 + 3x^4 - x^2y - x^2 - 7y^2x - 8$

Subtracting Polynomials

To subtract polynomials, change the signs in the polynomial being subtracted. Then add the two polynomials and combine similar terms.

EXAMPLE

Subtract. $(5x^2y + 3xy^2 - 7xy + 23) - (-6x^2y^2 - 8x^2y + 3xy + 7)$

Change the signs of the polynomial being subtracted.	$5x^2y + 3xy^2 - 7xy + 23 + 6x^2y^2 + 8x^2y - 3xy - 7$
Group similar terms.	$(5x^2y + 8x^2y) + 3xy^2 + (-7xy - 3xy) + 6x^2y^2 + (23 - 7)$
Combine similar terms.	$13x^2y + 3xy^2 - 10xy + 6x^2y^2 + 16$

Multiplying Polynomials

Polynomial by Monomial

Multiply each term of the polynomial by the monomial.

EXAMPLE

Multiply. $3x^2(7x^5y^3 - 4x + 3y - 2)$

$$\begin{array}{r} 7x^5y^3 - 4x + 3y - 2 \\ \times \qquad\qquad\quad 3x^2 \\ \hline 21x^7y^3 - 12x^3 + 9x^2y - 6x^2 \end{array}$$

Binomial by Binomial

This is the most common form of polynomial multiplication you will encounter. Multiply one binomial by each term of the other binomial. Then add similar terms.

EXAMPLE

Multiply. $(3x + 4)(6x - 7)$

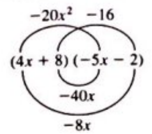
	$6x - 7$
	$\times\ 3x + 4$
Multiply by 4.	$24x - 28$
Multiply by $3x$.	$18x^2 - 21x$
Add similar terms.	$18x^2 + 3x - 28$

Notice that each binomial is a factor of the final product.

The FOIL Method

The diagram below shows how to multiply binomials quickly.

Multiply. $(4x + 8)(-5x - 2)$



Multiply the **F**irst terms.	$(4x)(-5x) = -20x^2$
Multiply the **O**uter terms.	$(4x)(-2) = -8x$
Multiply the **I**nner terms.	$(8)(-5x) = -40x$
Multiply the **L**ast terms.	$(8)(-2) = -16$

Combine similar terms.

$(4x + 8)(-5x - 2) = -20x^2 - 8x - 40x - 16 = -20x^2 - 48x - 16$

You can often multiply mentally.

Dividing Polynomials

Dividing polynomials is similar to regular division. The best way to understand division with polynomials is to work through an example.

EXAMPLE

$(x^3 + 4x^2 + 9x + 10) \div (x + 2) = ?$

Write the terms in each expression in descending order of exponent values.

$x + 2\overline{)x^3 + 4x^2 + 9x + 10}$

Focus on the leading term in the expression you are dividing by. In this example first ask, "how many times does x go into x^3?" The answer is x^2 times.

$$\begin{array}{l} \qquad\quad x^2 \\ x + 2\overline{)x^3 + 4x^2 + 9x + 10} \end{array}$$

5, 6, 7, 8 52–55
9, 10

Multiply $x^2(x + 2)$, write the partial product, and subtract. Just as in regular division, bring down the next term, $9x$.

$$\begin{array}{r} x^2 \\ x + 2 \overline{)\, x^3 + 4x^2 + 9x + 10} \\ \underline{-(x^3 + 2x^2)} \\ 2x^2 + 9x \end{array}$$

Divide $(x + 2)$ into the new term.

What is $2x^2$ divided by x? The answer is $2x$. Write:

$$\begin{array}{r} x^2 + 2x \\ x + 2 \overline{)\, x^3 + 4x^2 + 9x + 10} \\ \underline{-(x^3 + 2x^2)} \\ 2x^2 + 9x \end{array}$$

$2x(x + 2) = 2x^2 + 4x$, so subtract and bring down the next term:

$$\begin{array}{r} x^2 + 2x \\ x + 2 \overline{)\, x^3 + 4x^2 + 9x + 10} \\ \underline{-(x^3 + 2x^2)} \\ 2x^2 + 9x \\ \underline{-(2x^2 + 4x)} \\ 5x + 10 \end{array}$$

Now find $5x$ divided by x. The answer is 5.

Complete the problem.

$$\begin{array}{r} x^2 + 2x + 5 \\ x + 2 \overline{)\, x^3 + 4x^2 + 9x + 10} \\ \underline{-(x^3 + 2x^2)} \\ 2x^2 + 9x \\ \underline{-(2x^2 + 4x)} \\ 5x + 10 \\ \underline{-(5x + 10)} \\ 0 \end{array}$$

Therefore $(x^3 + 4x^2 + 9x + 10) \div (x + 2) = x^2 + 2x + 5$.

Multiply to check your answer.

$(x + 2)(x^2 + 2x + 5)$

$= x^3 + 2x^2 + 5x + 2x^2 + 4x + 10$

$= x^3 + 4x^2 + 9x + 10$

Dividing always works, but you may be able to:
- write as a fraction
- factor
- simplify

Divide $(x^2 + x - 6) \div (x + 3)$.

Write as a fraction

$$\frac{x^2 + x - 6}{x + 3}$$

Factor.

$$\frac{(x + 3)(x - 2)}{(x + 3)}$$

Simplify.

$$\frac{\cancel{(x + 3)}(x - 2)}{\cancel{(x + 3)}} = x - 2$$

MODEL ACT PROBLEM

1. $(2x^2 - 3y)(7y - x) = ?$
   - A. $2x^2 - 21xy + x$
   - B. $-2x^3 + 14x^2y + 3xy - 21y^2$
   - C. $-2x^3 + 14x^2y + 24xy$
   - D. $7x^2y - 23x^3y^2 + 3y$
   - E. $12xy - 21y^2 + 3xy$

SOLUTION

Multiply $(2x^2 - 3y)(7y - x)$.

$$\begin{aligned} (2x^2 - 3y)(7y - x) &= 14x^2y - 2x^3 - 21y^2 + 3xy \\ &= -2x^3 + 14x^2y + 3xy - 21y^2 \end{aligned}$$

The correct answer is B.

2. $(3x^3 + 6x) \div 3x = ?$
   - F. $x^2 + 2$
   - G. $x^2 - 2$
   - H. $-x^2 + 2$
   - J. $x^2 + 5$
   - K. $x^2 - 5x$

SOLUTION

$$\begin{array}{r} x^2 + 0 + 2 \\ 3x \overline{)\, 3x^3 + 0x^2 + 6x} \\ \underline{-(3x^3)} \\ 0 + 0x^2 \\ \underline{0} \\ 0 + 6x \\ \underline{-(\quad 6x)} \\ 0 \end{array}$$

The correct answer is F.

Practice

Combine similar terms.

1. $3x^2 + 4y + 3x^2y + 6y$
2. $7x + 3x^2y + 17x + 3yx^2 + 7x^2y^2$
3. $17x + 13 - 12xy + 16x^2 - 6y^2x + 4x^2y + 4xy$
4. $2x^2 - 3y + x^2 + 6y^2$
5. $15xy - 7x^2 + 9y^2 - 12xy + 2y^2 + 3y - 7x$

Add.

6. $(3x^5y - 2x^2 + 3y^2 - 12xy) + (7x^5y - 10y^2 - 8xy)$
7. $(5y^4 - 8y^3 + 2x^3 - 4x^2y^2 + 2y) + (12y^3 - 2x^3 + 14x^2y^2 - 19x)$
8. $(7x^2y^4 + 13xy^3 - 18y^2 + 2y) + (-21x^2y^4 - 17xy^3 + 5x^2 - 3y)$
9. $(8x^5 + 3x^2 - 5x^2y + 6xy^2) + (3x^5 - 2x^2 + 4x^3y - x^2y + 3xy^3 - 4y^2)$
10. $(6x^4 - 7x^3 + 2x^2y^2 - 4xy^2) + (6x^5 + 7x^4 + 3x^3 - 2x^2y^3 + 7xy^2)$

Subtract.

11. $(2x^5 - 4y^3 + 15x^2 - 6y) - (3x^5 + 2y^3 - 17y^2 + 6x)$
12. $(4xy^5 - 18x^5y + 3x^4 - 17y^3 + 11) - (5xy^5 - 12x^4 - 3y^3 + 12)$
13. $(11x^4 - 3x^3y^2 + 13xy - 12x) - (15x^4 + 17x^3y^2 - 15x^2y^3 + 19y)$
14. $(15x^8 + 9x^4 - 3x^2y + 5xy^3) - (15x^9 - 15x^4 - 3x^3y + x^2y)$
15. $(6x^2 - 3x + 3xy^2 + 3y^3 - 12) - (6x^3 - 3x^2 + 4xy^2 - 3y^5 - 18)$

QUESTION NUMBERS	PAGES TO STUDY
	44–48
1, 2, 3, 4	48–51
5, 6, 7, 8	52–55
9, 10	

Multiply.

16. $-5x^2(4x^3 - 3x + 4y^2 - 1)$
17. $8x^2(3x + 2xy + 4y - 6)$
18. $(3x + 6)(4x + 2)$
19. $(7x - 1)(2x + 5)$
20. $(9x + 4)(3x - 8)$

Divide.

21. $(2x^3 + 10x^2 - 8x) \div 2x$
22. $(3x^3 + 29x^2 + 9x - 6) \div (3x + 2)$
23. $(4x^4 + 8x^3 + 24x^2 + 4x) \div 4x$
24. $(x^3 + 10x^2 + 22x + 12) \div (x + 2)$
25. $(2x^3 - 4x^2 + 16x) \div 2x$

(Answers on page 350)

ACT-TYPE PROBLEMS

1. $(3x^5 - 2x^4y + 9x^2y^2 + 2xy) - (5x^4y^2 - 7x^5 - 18x^2y^2 + 2xy) = ?$
   - A. $4x^5 + 7x^4y - 9x^2y^2 + 4xy$
   - B. $10x^5 - 3x^4y^2 + 27x^2y^2$
   - C. $-5x^4y^2 + 10x^5 - 2x^4y + 27x^2y^2$
   - D. $-5x^2y^4 + 4x^5 + 9x^4y^4 + 4x^2y^2$
   - E. $-2xy^2 - 9x^5y - 27x^2y^2 + 4xy^6$

2. $5x^2y(2x^3 - 6x^2y^2 + 4xy^2) = ?$
   - F. $10x^6y - 30xy^4 + 20x^4y^4$
   - G. $10x^5y - 30x^4y^3 + 20x^3y^3$
   - H. $5x^5y - 5x^4y^3 + 5x^3y^3$
   - J. $2x^6y - 6x^4y^4 + 4x^3y^4$
   - K. $10x^6 - 30x^3y^3 + 20x^4y^3$

3. $(2x^2 + 5)(x^2 - 7) = ?$
   - A. $2x^4 - 14x^2 - 35$
   - B. $2x^4 + 5x^2 - 35$
   - C. $2x^4 - 9x^2$
   - D. $-9x^2 - 35$
   - E. $2x^4 - 9x^2 - 35$

4. $(2x^3 + 7x^2 - 11x - 7) \div (2x + 1) = ?$
   - F. $x^2 - 3x + 7$
   - G. $x^2 + 3x - 7$
   - H. $x^2 - 3x - 7$
   - J. $x^2 + 7x - 3$
   - K. $x^2 - 7x - 3$

5. $(2y^5x - 7y^3x^2 + 13y^2x - 6x) + (10y^5x + 3y^2x^3 - 21y^2x + 5x) = ?$
   - A. $12x^5y - 4y^2x^3 + 8yx^2 + 1$
   - B. $8y^5x - 4y^3x^2 + y^2x - x$
   - C. $12y^5x - 7y^3x^2 + 3y^2x^3 - 8y^2x - x$
   - D. $-12x^5y + 4y^2x^3 - 8yx^2 - 11x$
   - E. $-12y^5x - 7y^3x^2 - 3y^2x^3 + 8yx^2 + x$

(Answers on page 350)

Factoring Polynomials

Factors of a polynomial are expressions whose product equals the polynomial.

CALCULATOR TIP

The calculators allowed on the ACT cannot factor polynomials. You will have to use paper and pencil.

Factoring Out Common Factors

You may be able to find a common factor in each term of a polynomial.

- Choose the greatest common factor of the coefficients.
- Choose the smallest exponent for each variable.

EXAMPLES

1. Factor. $35x^7y^8 + 14x^2y^5 - 63x^8y^3 + 84x^5y^9$

   7 is the greatest common factor of the coefficients.

   x^2 and y^3 are the smallest powers of the variables.

   Factor out $7x^2y^3$.

   $35x^7y^8 + 14x^2y^5 - 63x^8y^3 + 84x^5y^9 = 7x^2y^3(5x^5y^5 + 2y^2 - 9x^6 + 12x^3y^6)$

2. Factor. $49z^3x - 24z^5x^3 + 10z^2x^6 - z^3x + 14z^2x^6$

   Combine similar terms. $48z^3x - 24z^5x^3 + 24z^2x^6$

   Factor out $24z^2x$. $24z^2x(2z - z^3x^2 + x^5)$

Factoring Completely

You may have to complete several steps before you have factored a polynomial completely.

EXAMPLES

1. Factor. $(4x - 6 - x + 10)(2x - 2) + (3x + 4)(5x - 1)$

   Combine similar terms. $(3x + 4)(2x - 2) + (3x + 4)(5x - 1)$

   Factor out $(3x + 4)$. $(3x + 4)[(2x - 2) + (5x - 1)]$

   Combine similar terms. $(3x + 4)(7x - 3)$

2. Factor. $12w^2 - 4wz + 21wz - 7z^2$

Group to show the two parts to factor. $(12w^2 - 4wz) + (21wz - 7z^2)$

Factor out $4w$ from one part and $7z$ from the other. $4w(3w - z) + 7z(3w - z)$

Factor out $(3w - z)$. $(4w + 7z)(3w - z)$

Special Factors

Memorize these methods of factoring polynomials. The first two are particular favorites of ACT test writers. Remember, these forms may appear either in polynomial form or in factored form. You will be asked to write them in the other form.

Difference of Squares

$x^2 - y^2 = (x + y)(x - y)$

EXAMPLES

1. Factor. $81x^2 - 64y^2$

   $81x^2 - 64y^2 = (9x + 8y)(9x - 8y)$

2. Factor. $144 - 0.25y^2$

   $144 - 0.25y^2 = (12 + 0.5y)(12 - 0.5y)$

You may have to factor the coefficients to reveal a special factor.
$3x^2 - 27$
$= 3(x^2 - 9)$
$= 3(x + 3)(x - 3)$

Perfect Square

$x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$

EXAMPLES

1. Factor. $16x^2 + 40xy + 25y^2$

   $16x^2 + 40xy + 25y^2 = (4x + 5y)(4x + 5y) = (4x + 5y)^2$

2. Factor. $4x^2 + 36x + 81$

   $4x^2 + 36x + 81 = (2x + 9)(2x + 9) = (2x + 9)^2$

Sum of Cubes

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

EXAMPLES

1. Factor. $8x^3 + 27y^3$

   $8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$

2. Factor. $64 + 8y^3$

   $64 + 8y^3 = (4 + 2y)(16 - 8y + 4y^2)$

Difference of Cubes

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

EXAMPLES

1. Factor. $125x^3 - y^3$

   $125x^3 - y^3 = (5x - y)(25x^2 + 5xy + y^2)$

2. Factor. $0.008x^3 - 27$

   $0.008x^3 - 27 = (0.2x - 3)(0.04x^2 + 0.6x + 9)$

MODEL ACT PROBLEM

1. Factor. $12x^4y + 18x^3y^2 - 30x^2y$

   A. $4xy(3x^3y + 4x^2y^2 - 7x^2y)$
   B. $6x^2y(2x^2 + 3xy - 5)$
   C. $6xy(2x^2 + 3x^2y - 5y)$
   D. $3x^2y(4x^2 + 6xy - 10)$
   E. $3x^2y(2x^2 + 3x^2y - 5y)$

SOLUTION

Choose the greatest common factor of the coefficients. 6

Choose the smallest exponent of each variable. x: x^2

y: y

Factor the polynomial.

$12x^4y + 18x^3y^2 - 30x^2y = 6x^2y(2x^2 + 3xy - 5)$

The correct answer is B.

2. Factor. $2x^2 - 2x + 3x - 3$

   F. $(x - 1)(x + 3)$
   G. $(x - 3)(2x + 1)$
   H. $(x - 1)(2x + 3)$
   J. $(x + 3)(x - 1)$
   K. $(2x - 1)(x + 3)$

SOLUTION

Group to show the parts to be factored.

$(2x^2 - 2x) + (3x - 3)$

Factor out $2x$ from the first part and 3 from the second part.

$2x(x - 1) + 3(x - 1)$

Factor out $(x - 1)$ because it is the common factor in both terms.

$(x - 1)(2x + 3)$

The correct answer is H.

Practice

Factor each polynomial completely.

1. $4x^4 + 10x^4y^2 - 2x^4y^2 + 2x^4$
2. $4x^2y^3 - 12y^4 + 16xy^4$
3. $12x^3 + 10x^2 - 20x$
4. $z^2 + 8w^2 - 1.49z^2 + 17w^2$
5. $17k^2 + 10y^2 - 2k^2 - 5y^2$
6. $27z^2x^2 - 12z^2x^2 + 9z^2$
7. $5x^3y^2 + 10xy - 15x^2y^2 - 5xy^2$
8. $8x^4y^5 - 16x^2y + 12x^3y^3$
9. $x^2 + 3x - 5x - 15$
10. $72x^4y^2z^4 + 60xyz$
11. $7y^2z - 28yz - 21y^2z + 14yz$
12. $4x^2 - 16y^2$
13. $6x^2 + 15x - 14x - 35$
14. $7x^3z^2 - 13x^2z^2 + 27x^2z$
15. $64x^3 - 27y^3$
16. $16x^4z^3 - 4x^3z^2 - 8x^2z^2 + 20x^5z^3$
17. $2x^2 + 12x - 9x - 54$
18. $(4x - 7 - 2x + 2)(3x - 4) + (2x - 5)(3x + 1)$
19. $125x^3 + 343y^3$
20. $x^2 - x + 2x - 2$

(Answers on page 350)

ACT-TYPE PROBLEMS

1. Factor $144x^2y^2 - 169z^2$ completely.
   A. $(12xy - 13z)(12xy + 13z)$
   B. $(12x - 13y)(12x + 13y)$
   C. $(13xz + 12y)(13xz - 12y)$
   D. $(12xz - 13y)(12xz + 13y)$
   E. $(12 - 13z)(12y + 13z)$

2. Factor $6x^2 + 27x - 14x - 63$ completely.
   F. $(7x - 3)(9x + 2)$
   G. $(2x + 9)(3x - 7)$
   H. $(7x + 9)(3x - 2)$
   J. $(7x + 3)(9x - 2)$
   K. $(9x + 3)(7x - 2)$

3. Factor $24x^3y^2 - 12x^2y^2 + 16x^3y^3 - 8x^5y^4$ completely.
   A. $8x^2y^2(3x - 2 + 2xy - x^3y^2)$
   B. $2xy(12x^2y - 6xy + 8x^2y^2 - 4x^4y^3)$
   C. $4x^2y^2(6x - 3 + 4xy - 2x^3y^2)$
   D. $4xy(6x^2y - 3xy + 4x^2y^2 - 2x^4y^3)$
   E. $2x^2y^2(12x - 4 + 8xy - 4x^3y^2)$

4. Factor $(3y - 5 - 2y + 3)(2y + 3) + (y - 2)$.
   F. $(2y - 1)(y + 4)$
   G. $(4y - 1)(y - 1)$
   H. $2(y - 2)(y + 2)$
   J. $2(y^2 - 4)$
   K. $2(y - 1)(y + 2)$

5. When $9x^2 - 16$ is completely factored, what is the sum of the factors?
   A. $3x - 4$
   B. $3x + 4$
   C. $3x$
   D. $6x$
   E. $3x + 8$

(Answers on page 351)

■ Quadratic Equations

Quadratic equations can be written in the form $ax^2 + bx + c = 0$ $(a \neq 0)$. They can also be written in function notation: $f(x) = ax^2 + bx + c$ $(a \neq 0)$.

Factors of the quadratic equation are expressions whose product is $ax^2 + bx + c$.

Factoring Polynomials in the Form $ax^2 + bx + c$ $(a \neq 0)$

Usually, factoring quadratic expressions begins with an educated guess.

EXAMPLES

1. Factor. $x^2 + 4x - 5 = 0$

   The first terms must both be $(x$ ___$)(x$ ___$)$.

   The second terms must be -5 and 1 or -1 and 5 because the product must be -5.

   Adding -1 and 5 gives 4, the coefficient of the middle term.

   $x^2 + 4x - 5 = (x - 1)(x + 5)$

2. Factor. $10x^2 - 6x = 5x + 6$

   Always write the equation as a quadratic equal to 0.

   $10x^2 - 6x = 5x + 6 \rightarrow 10x^2 - 11x - 6 = 0$

   Then factor the trinomial $10x^2 - 11x - 6$.

   The first terms could be $(x$ ___$)(10x$ ___$)$ or $(2x$ ___$)(5x$ ___$)$.

   The product of the second terms must be -6.

   So the possible factors are:

$(x$ ___$)$	$(10x$ ___$)$	or	$(2x$ ___$)$	$(5x$ ___$)$
-6	1		-6	1
6	-1		6	-1
1	-6		1	-6
-1	6		-1	6
-3	2		-3	2
3	-2		3	-2
-2	3		-2	3
2	-3		2	-3

   The sum of the inner and outer products must be $-11x$.

   $-15x$

   $(2x - 3)(5x + 2)$

   $4x$

   $10x^2 - 11x - 6 = 0$

   $(2x - 3)(5x + 2) = 0$

Sometimes you may find special factors.

EXAMPLES

1. Factor. $16x^2 + 40x + 25 = 0$

   This polynomial is a perfect square.

   $16x^2 + 40x + 25 = (4x + 5)(4x + 5) = (4x + 5)^2$

2. Factor. $36x^2 - 49 = 0$

   This polynomial is the difference of squares.

   $36x^2 - 49 = (6x + 7)(6x - 7)$

Solving Quadratic Equations by Factoring

Follow these steps.

- Write the equation in the form $ax^2 + bx + c = 0$.
- Factor the polynomial $ax^2 + bx + c$.
- Find the solution set.

The solution set contains the roots of the polynomial.

5, 6, 7, 8

CALCULATOR TIP

Many calculators can solve any equation, including quadratic equations. Use the method that works on your calculator.

EXAMPLES

1. Factor to solve the quadratic equation. $x^2 + 4x - 5 = 0$

$x^2 + 4x - 5 = (x - 1)(x + 5)$

Since $x^2 + 4x - 5 = 0$, $(x - 1)(x + 5) = 0$.

If $(x - 1)(x + 5) = 0$, then $(x - 1) = 0$ or $(x + 5) = 0$ or they both equal zero.

Solve the equations $x - 1 = 0$ and $x + 5 = 0$ to find the solution set for the quadratic equation.

$$x - 1 = 0 \qquad x + 5 = 0$$
$$x = 1 \qquad x = -5$$

The solution set for the equation $x^2 + 4x - 5 = 0$ is $\{1, -5\}$.

2. Solve the quadratic equation. $10x^2 - 6x = 5x + 6$

Write in standard form. $10x^2 - 6x = 5x + 6 \rightarrow 10x^2 - 11x - 6 = 0$

$10x^2 - 11x - 6 = (2x - 3)(5x + 2)$

So $(2x - 3)(5x + 2) = 0$.

If $(2x - 3)(5x + 2) = 0$, then $(2x - 3) = 0$ or $(5x + 2) = 0$ or they both equal zero.

Solve the equations $2x - 3 = 0$ and $5x + 2 = 0$ to find the solution set for the quadratic equation.

$$2x - 3 = 0 \qquad 5x + 2 = 0$$
$$2x = 3 \qquad 5x = -2$$
$$x = \frac{3}{2} \qquad x = -\frac{2}{5}$$

The solution set for the equation $10x^2 - 11x - 6 = 0$ is $\left\{\frac{3}{2}, -\frac{2}{5}\right\}$.

3. Solve the quadratic equation. $16x^2 + 40x + 25 = 0$

$$16x^2 + 40x + 25 = (4x + 5)^2$$
$$4x + 5 = 0$$
$$4x = -5$$
$$x = -\frac{5}{4}$$

The solution set is $\left\{-\frac{5}{4}\right\}$.

4. Solve the quadratic equation. $36x^2 - 49 = 0$

$36x^2 - 49 = (6x + 7)(6x - 7)$

$$6x + 7 = 0 \qquad 6x - 7 = 0$$
$$6x = -7 \qquad 6x = 7$$
$$x = -\frac{7}{6} \qquad x = \frac{7}{6}$$

The solution set is $\left\{-\frac{7}{6}, \frac{7}{6}\right\}$.

MODEL ACT PROBLEMS

1. What is the solution set for the quadratic equation $16x^2 - 4 = 0$?

A. $\left\{\frac{1}{2}, -\frac{1}{2}\right\}$

B. $\{2, -2\}$

C. $\{0\}$

D. $\{1, -1\}$

E. $\left\{\frac{1}{3}, -\frac{1}{3}\right\}$

SOLUTION

The equation $16x^2 - 4 = 0$ is the difference of two squares.

$(4x - 2)(4x + 2) = 0$

Set each factor equal to 0 to find the solution set.

	$4x - 2 = 0$		$4x + 2 = 0$
Add 2.	$4x = 2$	Subtract 2.	$4x = -2$
Divide by 4.	$x = \frac{1}{2}$	Divide by 4.	$x = -\frac{1}{2}$

$x = \left\{\frac{1}{2}, -\frac{1}{2}\right\}$

The correct answer is A.

Hint:
You can also substitute answers in the equation. Keep trying answers until you find an answer set that makes the equation correct.

2. What is the solution set to the quadratic equation $2x^2 + 3x = 2$?

F. $\{1, -1\}$

G. $\{2, -2\}$

H. $\left\{\frac{1}{2}, -\frac{1}{2}\right\}$

J. $\left\{\frac{1}{2}, -2\right\}$

K. $\left\{-\frac{1}{2}, 2\right\}$

SOLUTION

Rewrite the equation in standard form.
$2x^2 + 3x = 2 \rightarrow 2x^2 + 3x - 2 = 0$

The first terms must be: $(2x$ ____$)(x$ ____$) = 0$

The product of the outer terms must be -2, so the possible factors are:

$(2x$ ____$)(x$ ____$) = 0$

−1	2
2	−1
1	−2
−2	1

The sum of the inner and outer products must be 3, so the correct factorization is $(2x - 1)(x + 2) = 0$.

Solve the quadratic equation.

	$2x - 1 = 0$		$x + 2 = 0$
Add 1.	$2x = 1$	Subtract 2.	$x = -2$
Divide by 2.	$x = \frac{1}{2}$		

The solution set is $x = \left\{\frac{1}{2}, -2\right\}$.

The correct answer is J.

1, 2, 3, 4
5, 6, 7, 8

44–48
48–51

Practice

Factor to solve each quadratic equation.

1. $x^2 + 4x + 3 = 0$
2. $4x^2 - 10x + 6 = 0$
3. $9x^2 + 54x = -81$
4. $6x^2 + x = 15$
5. $7x^2 = 126$
6. $8x^2 + 40x = 0$
7. $x = -2x^2 + 21$
8. $4x^2 = 16$
9. $9x^2 + 12x + 4 = 0$
10. $-19x - 5 = -4x^2$
11. $x^2 - x - 6 = 0$
12. $25x^2 - 36 = 0$
13. $0 = -x^2 + 9$
14. $3x^2 = x + 14$
15. $4x^2 - 20x = -25$
16. $x^2 + 24 = -14x$
17. $14x^2 - 5x - 1 = 0$
18. $2x^2 = -19x - 39$
19. $x^2 + 2x + 1 = 0$
20. $29x = -10x^2 + 21$

(Answers on page 351)

ACT-TYPE PROBLEMS

1. What is the solution set to the quadratic equation $x^2 - 81 = 0$?

A. {1, −1}
B. {3, −3}
C. {5, −5}
D. {7, −7}
E. {9, −9}

2. What is the solution set to the quadratic equation $4x^2 - 24x = -36$?

F. {3}
G. {3, −3}
H. {2}
J. {2, −2}
K. {−3}

3. Which of the following cannot be a solution set for a quadratic equation?

A. {15}
B. {−1, 1, 2}
C. {−5, 5}
D. {3}
E. {3, 7}

4. What is the sum of the solutions to the quadratic equation $(x - 3)(x + 5) = 0$?

F. 8
G. 2
H. 5
J. −2
K. 3

5. What is the product of the solutions to the quadratic equation $10x^2 = -21x + 10$?

A. −2
B. −1
C. 0
D. 1
E. 2

(Answers on page 351)

Remember, it is acceptable to work backwards from the answer. Keep substituting until the equation "works out."

Cumulative ACT Practice

Elementary Algebra

Complete this Cumulative ACT Practice in 10 minutes to reflect real ACT test conditions. This Cumulative ACT Practice gives you an additional opportunity to practice elementary algebra concepts in an ACT format. If you don't know an answer, eliminate and guess. Circle the number of any guessed answer. Then check your answers on page 352. You will also find explanations for the answers and suggestions for further study.

1. In the formula $E = IR$, E = Voltage, I = Amperage, and R = Resistance. If the voltage is 12 what must the amperage be so that the resistance is 2.5?

A. 4
B. 4.5
C. 4.8
D. 5.3
E. 5.8

2. $x = 2$ and $x = -4$ is the solution set for which of the following quadratic equations?

F. $x^2 - 2x + 8 = 0$
G. $x^2 + 2x - 8 = 0$
H. $x^2 - 8x + 2 = 0$
J. $x^2 + 8x + 2 = 0$
K. $x^2 - 8x - 2 = 0$

3. $3\sqrt{15}$ is the simplified form of which expression?

A. $\frac{30\sqrt{3}}{2\sqrt{5}}$
B. $3\sqrt{10} + 3\sqrt{5}$
C. $\sqrt{45} - \sqrt{30}$
D. $\sqrt[3]{5} \cdot \sqrt[3]{3} \cdot 3$
E. $45 \div 3\sqrt{15}$

4. $5a^2b^3 - 2ab - 3a + 3b$ is formed by adding which two polynomials?

F. $4a^2b^3 - 5ab + 3a$ and $2a^2b^2 - 3ab - 2b$
G. $a^2b^3 + 2ab - 3a$ and $4a^2b^3 - 4ab + 3b$
H. $5a^2b^3 + 2a - 3b$ and $-2ab + 5a$
J. $a^3b^2 + 2ab + 3a$ and $4a^3b^2 + 4ab - 3b$
K. $-a^2b^2 + 2ab - 3a$ and $4a^2b^3 - 4ab + 3b$

5. $\frac{7x^3 + 9x^2 + 11x - 15}{7x - 5} = ?$

A. $x^2 + 2x + 3$
B. $2x^2 - 3x + 5$
C. $x^2 - 2x - 3$
D. $2x^2 + 2x + 3$
E. $x^2 - 3x + 5$

6. What is the sum of the solutions of the equation $x^2 - 5x + 6 = 0$?

F. −5
G. −4
H. 4
J. 5
K. 6

7. $2^5 \div \sqrt[3]{64} = ?$

A. 16
B. 8
C. 4
D. 2.5
E. 2

8. What is the value of $\frac{3^3 \div 3 - 6}{\left(\frac{1}{2}\right)^{-2} - 1}$?

F. 1
G. 2
H. 3
J. 4
K. 5

9. In simplest form, $5\sqrt{12} + 7\sqrt{108} =$

A. $26\sqrt{12}$
B. $12\sqrt{120}$
C. $52\sqrt{6}$
D. $52\sqrt{3}$
E. $20\sqrt{3}$

10. What is the value of $\frac{(\sqrt[3]{8} - 2) + 3}{(-2)^{-1} + 1}$?

F. 7
G. 6
H. 3
J. 2
K. 1

MODEL ACT PROBLEMS

1. What is the solution set to the inequality $2x + 5 \geq 13$?

   A. $x \leq 9$
   B. $x \geq 9$
   C. $x \leq 4$
   D. $x \geq 4$
   E. $x \leq -4$

SOLUTION

$$2x + 5 \geq 13$$

Subtract 5. $\quad 2x \geq 8$

Divide by 2. $\quad x \geq 4$

The correct answer is D.

2. What is the smallest number in the solution set to the inequality $-4x - 5 \leq -2x + 9$?

   F. -14
   G. -7
   H. 0
   J. 7
   K. 14

SOLUTION

$$-4x - 5 \leq -2x + 9$$

Add 5. $\quad -4x \leq -2x + 14$

Add $2x$. $\quad -2x \leq 14$

Divide by -2. $\quad x \geq -7$

The correct answer is G.

Practice

Solve each inequality.

1. $x + 9 > 13$
2. $-13 + x \leq 22$
3. $-3y \leq 7$
4. $-\left(\frac{y}{6}\right) > 11$
5. $-4t + 6 \leq 9$
6. $\frac{k}{8} - 2.4 > 12$
7. $-5x + 6 \leq 2x - 8$
8. $7 + 4t < 3t - 2$
9. $2x + 5 < 4$
10. $-3x > 9$
11. $2x - 7 \geq 12$
12. $7x - 3 < 3x + 1$
13. $-2x + 9 \leq 13$
14. $16x - 7 \geq 6x + 2$
15. $-4x + 9 \geq 27$
16. $-3x + 7 > 11x - 2$
17. $-x - 13 > 2$
18. $27x - 5 < 35x + 3$
19. $9x - 5 \leq 6x + 2$
20. $x + 3 < 2x - 1$

(Answers on page 359)

ACT-TYPE PROBLEMS

1. What is the solution set for the inequality $4x \geq -12$?

   A. $x \geq -4$
   B. $x \leq -3$
   C. $x \geq -3$
   D. $x \geq 3$
   E. $x \leq 4$

2. What is the solution set to the inequality $-3x - 7 < 20$?

   F. $x > 13$
   G. $x < 13$
   H. $x < 9$
   J. $x > 9$
   K. $x > -9$

3. What is the solution set to the inequality $-4x + 17 \leq -3$?

   A. $x \leq -5$
   B. $x \geq -4$
   C. $x \leq 4$
   D. $x \geq 5$
   E. $x \leq 5$

4. Which of the following is not in the solution set to the inequality $-5x + 3 > -2x - 12$?

   F. 5
   G. 4
   H. 3
   J. 2
   K. 1

5. Which of the following is the reciprocal of the smallest number in the solution set to the inequality $-9x + 11 \leq -4x + 3$?

   A. -40
   B. $-\frac{8}{5}$
   C. $-\frac{5}{8}$
   D. $\frac{5}{8}$
   E. $\frac{8}{5}$

(Answers on page 360)

Absolute Value Equations and Inequalities

When solving absolute value equalities and inequalities you must consider two possibilities.

For example: If $|x| = 7$, then $x = 7$ or $x = -7$.

To solve absolute value equations and inequalities you must solve for each case.

Case 1: The value is positive. Drop the absolute value and solve.

Case 2: The value is negative. Drop the absolute value. Use a minus sign to make the expression from inside the absolute value negative and solve.

EXAMPLES

1. $|x - 8| = 5$

   Case 1: $x - 8$ is positive $\quad x - 8 = 5 \quad x = 13$

   Case 2: $x - 8$ is negative $\quad -(x - 8) = 5 \quad -x + 8 = 5 \quad -x = -3 \quad x = 3$

   $x = 3$ or $x = 13$

   Check: $|13 - 8| = |5| = 5$

   $|3 - 8| = |-5| = 5$

2. $|x + 4| < 7$

   Case 1: $x + 4 < 7 \quad x < 3$

   Case 2: $-(x + 4) < 7 \quad -x - 4 < 7 \quad -x < 11 \quad x > -11$

   Therefore $-11 < x < 3$.

   Check: Check a sample of the values between -11 and 3. Each value makes the original inequality correct.

MODEL ACT PROBLEM

Which of the following is the largest number that will make the inequality $|2x - 8| \le 6$ true?

A. 7
B. 5
C. 3
D. 1
E. 0

SOLUTION

Solve the inequality.

$2x - 8 \le 6 \rightarrow 2x \le 14 \rightarrow x \le 7$

$-(2x - 8) \le 6 \rightarrow -2x + 8 \le 6 \rightarrow -2x \le -2 \rightarrow -x \le -1 \rightarrow x \ge 1$

$1 \le x \le 7$

The correct answer is A.

Practice

Solve.

1. $|3x - 4| < 14$
2. $|x - 6| \le 9$
3. $|x + 3| = 11$
4. $|5x - 12| \ge 13$
5. $|2x + 8| > 12$
6. $|3x - 3| = 12$
7. $|6x + 5| > 9$
8. $|x - 5| \ge 2$
9. $|3x - 1| < 5$
10. $|2x + 4| \le 6$
11. $|x - 3| < 1$
12. $|2x + 7| = 9$
13. $|7x - 3| \ge 7$
14. $|5x + 4| \le 4$
15. $|4x - 9| > 3$
16. $|3x + 3| = 4$
17. $|8x - 5| \le 9$
18. $|2x + 3| < 12$
19. $|5x - 5| \ge 7$
20. $|x + 1| > 14$

(Answers on page 360)

ACT-TYPE PROBLEMS

1. 16 and −10 are the solutions to which one of the following equations?

   A. $|x - 2| = 14$
   B. $|x - 2| = 8$
   C. $|x + 2| = 12$
   D. $|x - 3| = 13$
   E. $|x + 3| = 13$

2. Which of the following choices makes the inequality $|2x - 9| < 5$ false?

   F. 2
   G. 3
   H. 4
   J. 5
   K. 6

3. Solve the inequality $|7x - 5| \ge 9$.

   A. $-4 \le x \le 2$
   B. $x \le \frac{4}{7}$ or $x \ge 2$
   C. $x \le -\frac{4}{7}$ or $x \ge 2$
   D. $x \le -\frac{4}{7}$ or $x \ge -2$
   E. $-\frac{4}{7} \le x \le 2$

4. $x = 6$ is the complete solution set to which of the following equations?

   F. $2x = 12$
   G. $|x - 3| = 3$
   H. $|2x - 2| = 10$
   J. $-x = 6$
   K. $|x + 6| = 12$

5. What is the product of the solutions to the equation $|2x - 3| = 15$?

   A. 45
   B. 9
   C. −6
   D. −54
   E. −81

(Answers on page 360)

Solving Systems of Linear Equations

A linear equation is any equation in the form $ax + by = c$ ($a \ne 0, b \ne 0$). The solution to a linear equation is an ordered pair (x,y) that makes the equation true.

A system of linear equations is two or more linear equations that can be solved together. The solution to a system of linear equations must be the solution for all of the equations in the system.

To solve a system of equations, sometimes you can add or subtract the equations to eliminate one of the variables. Other times you will have to change an equation so that when you add or subtract, one of the terms is eliminated.

The solution may be:
- an ordered pair
- the entire line
- no solution (lines are parallel)

EXAMPLES

1. Solve. $3x + 5y = 16$
   $-3x + 3y = 8$

   This one is easy. Add the two equations.

   Add.
   $$\begin{aligned} 3x + 5y &= 16 \\ -3x + 3y &= 8 \\ \hline 8y &= 24 \end{aligned}$$

   Solve for y.
   $$y = 3$$

   Substitute 3 for y in one of the equations.
   $$3x + 5(3) = 16$$

   Solve for x.
   $$\begin{aligned} 3x + 15 &= 16 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$

   The solution to the system is $x = \frac{1}{3}$ and $y = 3$.

   The solution as an ordered pair is $\left(\frac{1}{3}, 3\right)$.

Substitution

You can also:
- Solve one equation
- Substitute the result in the other equation
- Solve that equation

CALCULATOR TIP

Graphing calculators can be used to graph and solve systems of linear equations.

2. Solve. $-4x - 4y = 8$
$2x + 7y = 15$

Multiply the bottom equation by 2. $2(2x + 7y = 15) \rightarrow 4x + 14y = 30$

$$\begin{aligned} -4x - 4y &= 8 \\ 4x + 14y &= 30 \\ \hline 10y &= 38 \\ y &= 3.8 \end{aligned}$$

Add the equations.

Solve for y.

Substitute 3.8 for y in one of the equations.

Solve for x.

$$\begin{aligned} -4x - 4(3.8) &= 8 \\ -4x &= 23.2 \\ 4x &= -23.2 \\ x &= -5.8 \end{aligned}$$

The solution is $x = -5.8$ and $y = 3.8$.

The solution as an ordered pair is (-5.8,3.8).

3. Solve. $-8y + 5x + 12 = 2$
$6x + 12y = 6$

Rewrite the equations in linear form.

$$\begin{aligned} 5x - 8y &= -10 \\ 6x + 12y &= 6 \end{aligned}$$

Multiply the top equation by 1.5. $1.5(5x - 8y = -10) \rightarrow 7.5x - 12y = -15$

Add the equations.

Solve for x.

$$\begin{aligned} 7.5x - 12y &= -15 \\ 6x + 12y &= 6 \\ \hline 13.5x &= -9 \\ x &= -\frac{2}{3} \end{aligned}$$

Substitute $-\frac{2}{3}$ for x in one of the equations.

Solve for y.

$$\begin{aligned} 6\left(-\frac{2}{3}\right) + 12y &= 6 \\ -4 + 12y &= 6 \\ 12y &= 10 \\ y &= \frac{5}{6} \end{aligned}$$

The solution is $x = -\frac{2}{3}$ and $y = \frac{5}{6}$.

The solution as an ordered pair is $\left(-\frac{2}{3}, \frac{5}{6}\right)$.

> Change the equations so that when you add or subtract the equations, you "get rid" of one of the variables.

> Rewrite the equations so that you can add or subtract.

MODEL ACT PROBLEMS

1. What is the solution to the system of linear equations $2x + 5y = 10$ and $2x + 3y = 2$?

   A. (−5,−4)
   B. (−5,4)
   C. (−4,−5)
   D. (4,−5)
   E. (5,4)

SOLUTION

Subtract one equation from the other.

Solve for y.

$$\begin{aligned} 2x + 5y &= 10 \\ -2x - 3y &= -2 \\ \hline 2y &= 8 \\ y &= 4 \end{aligned}$$

Substitute 4 for y in one of the equations.

Solve for x.

$$\begin{aligned} 2x + 5(4) &= 10 \\ 2x + 20 &= 10 \\ 2x &= -10 \\ x &= -5 \end{aligned}$$

The solution is $x = -5$ and $y = 4$.

The solution as an ordered pair is (−5,4).

The correct answer is B.

2. What is the sum of the solutions to the following system of linear equations?

$$\begin{aligned} 6x - 5y &= 15 \\ -3x + 2y &= 10 \end{aligned}$$

   F. $-61\frac{2}{3}$
   G. −35
   H. $8\frac{1}{3}$
   J. 25
   K. $26\frac{2}{3}$

SOLUTION

Multiply both sides of the second equation by 2. $2(-3x + 2y = 10) \rightarrow -6x + 4y = 20$

Add the equations.

$$\begin{aligned} 6x - 5y &= 15 \\ -6x + 4y &= 20 \\ \hline -y &= 35 \\ y &= -35 \end{aligned}$$

Substitute −35 for y in one of the equations.

Solve for x.

$$\begin{aligned} -3x + 2(-35) &= 10 \\ -3x - 70 &= 10 \\ -3x &= 80 \\ x &= -\frac{80}{3} = -26\frac{2}{3} \end{aligned}$$

The solutions are $x = -26\frac{2}{3}$ and $y = -35$.

Find the sum of the solutions. $-26\frac{2}{3} + (-35) = -61\frac{2}{3}$

The correct answer is F.

Practice

Solve the system of equations.

1. $4x + 5y = 13$
   $4x + 3y = 9$

2. $3x - 2y = 6$
   $9x + 6y = 60$

3. $-5y + 3x = 8\frac{3}{4}$
   $2y + 12x + 13 = 26$

4. $2x + 5y = 6$
   $2x + 4y = 5$

5. $3x + 5y = 7$
   $6x + 5y = 2$

6. $4x - 9y = 8$
   $4x + 9y = 8$

7. $2x + 4y = 9$
   $3x - 4y = 8$

8. $x + 3y = 5$
   $2x + 4y = 6$

9. $4x + 5y = 8$
   $6x + 8y = 7$

10. $5x + 12y = 13$
    $3x + 4y = 5$

11. $12x + 8y = 2$
    $4x + 6y = 10$

12. $-6x + 7y = -13$
    $-12x + 8y = 4$

13. $x - 5y = 10$
    $-2x + 3y = 8$

14. $-12x + 8y = -5$
    $-4x + 6y = -5$

15. $-4x + 9y = 19$
    $-6x + 11y = -20$

16. $-2x + 5y = 6$
    $-3x + 4y = 9$

17. $9x + 7y = 5$
    $8x + 6y = 4$

18. $-16x + 7y = 5$
    $17x - 8y = 2$

19. Holly has 15 dimes and nickels worth $1.05. How many of each type of coin does Holly have?

20. Suresh studied 8 hours for his final exams in math and science. He studied 1.5 hours longer for his math final than for his science final. How many hours did he study for each final?

21. Tia went to a sale at a media store. She bought 8 videos and CDs for $92. If each video cost $16 and each CD cost $10, how many of each did she buy?

(Answers on page 360)

ACT-TYPE PROBLEMS

1. What is the solution to the following system of linear equations?

   $2x + 5y = 8$

   $2x + 4y = 7$

   A. (1.5,1)
   B. (1,1.5)
   C. (1,−1.5)
   D. (−1.5,1)
   E. (−1.5,−1)

2. What is the solution to the following system of linear equations?

   $3x + 5y = 8$

   $-3x + 5y = 8$

   F. (1.6,0)
   G. (0,1.6)
   H. (0,1.4)
   J. (0,−1.6)
   K. (−1.4,0)

3. What is the product of the solutions to the following system of linear equations?

   $7x + 10y = 12$

   $5x + 5y = 6$

   A. −1.2
   B. −1
   C. 0
   D. 1
   E. 1.2

4. You are to find two numbers. When you double the first and triple the second, their sum is 1. When you triple the first and multiply the second by 5, the sum is 2. What are the two numbers?

   F. (1,0.5)
   G. (1,−1)
   H. (−0.5,1)
   J. (−1,1)
   K. (−1,−1)

5. There are two paths. In the morning, 6 people walked the first path and 12 people walked the second path. The total distance these people walked was 8 miles. In the afternoon, 9 people walked the first path and 4 people walked the second path. The total distance people walked in the afternoon was 5 miles. How many miles long is each path?

   A. $\left(\frac{1}{2}\text{ mile}, \frac{1}{3}\text{ mile}\right)$
   B. $\left(\frac{1}{3}\text{ mile}, \frac{1}{2}\text{ mile}\right)$
   C. (3 miles,2 miles)
   D. (2 miles,3 miles)
   E. (5 miles,8 miles)

(Answers on page 360)

Rational and Radical Expressions

Simplifying Expressions

To simplify rational and radical expressions, you may need to use some or all of these equalities.

$\sqrt[n]{x}$ means the nth root of x.

$$\sqrt[a]{x^b} = x^{\left(\frac{b}{a}\right)}$$

$$x^{(-a)} = \frac{1}{x^a} \quad \left(\frac{1}{x^{(-a)}} = x^a\right)$$

$$\sqrt{x} \cdot \sqrt{x} = x$$

When the bases are the same, use these equalities to multiply and divide exponents.

$$x^b \cdot x^a = x^{(b+a)} \qquad x^b \div x^a = x^{(b-a)}$$

Undefined Expressions

An expression is considered undefined when its denominator is equal to zero, or any time there is division by zero. Otherwise, the expression is defined.

$\frac{4\sqrt{3}}{x}$ is defined except when $x = 0$.

$\frac{x^{\left(\frac{1}{3}\right)}}{3^x}$ is defined for all values of x.

$\frac{4(2x + 8)}{x - 8}$ is defined for all values of x except $x = 8$.

EXAMPLES

1. Simplify. $\dfrac{11x}{\sqrt{3x-8}}$

$\dfrac{11x}{\sqrt{3x-8}}$

$= \dfrac{11x}{\sqrt{3x-8}} \cdot \dfrac{\sqrt{3x-8}}{\sqrt{3x-8}}$ Multiply numerator and denominator by $\sqrt{3x-8}$. This removes the radical from the denominator.

$= \dfrac{11x\sqrt{3x-8}}{3x-8}$

2. Simplify. $\dfrac{1}{x^{\left(-\frac{2}{3}\right)}} + \dfrac{\sqrt[3]{x^2}}{x^{\left(-\frac{1}{3}\right)}}$

$\dfrac{1}{x^{\left(-\frac{2}{3}\right)}} + \dfrac{\sqrt[3]{x^2}}{x^{\left(-\frac{1}{3}\right)}}$

$= x^{\left(\frac{2}{3}\right)} + x^{\left(\frac{1}{3}\right)} \cdot \sqrt[3]{x^2}$ Use $x^{(-a)} = \dfrac{1}{x^a}$ $\left(\dfrac{1}{x^{(-a)}} = x^a\right)$

$= x^{\left(\frac{2}{3}\right)} + x^{\left(\frac{1}{3}\right)} \cdot x^{\left(\frac{2}{3}\right)}$ Use $\sqrt[a]{x^b} = x^{\left(\frac{b}{a}\right)}$

$= x^{\left(\frac{2}{3}\right)} + x^{\left(\frac{1}{3}+\frac{2}{3}\right)}$ Use $x^b \cdot x^a = x^{(b+a)}$

$= x^{\left(\frac{2}{3}\right)} + x^{\left(\frac{3}{3}\right)}$ Add fractional exponents with the same base.

$= x^{\left(\frac{2}{3}\right)} + x$ Simplify a fractional exponent. $x^{\left(\frac{3}{3}\right)} = x^1 = x$

3. For which real values of x is $\dfrac{7x}{2^{(4-x)} - 8}$ defined?

Find the values of x for which the expression is *not* defined.

$\dfrac{7x}{2^{(4-x)} - 8}$ is not defined when $2^{(4-x)} - 8 = 0$.

$2^{(4-x)} - 8 = 0$ when $2^{(4-x)} = 8$.

$2^{(4-x)} = 8$ when $x = 1$. $(2^{(4-1)} = 2^3 = 8)$

The expression is *not* defined when $x = 1$.

The expression *is* defined for all real values of x except $x = 1$.

MODEL ACT PROBLEM

Write the expression $\dfrac{\sqrt{x^3}+x}{\sqrt{x}}$ in simplified form with no radicals and no negative exponents. $(x \neq 0)$

A. x

B. $x^{\left(\frac{5}{2}\right)}$

C. $\dfrac{x+1}{x^{\left(\frac{1}{2}\right)}}$

D. $x + x^{\left(\frac{1}{2}\right)}$

E. $x^2 + \dfrac{1}{x}$

SOLUTION

$\dfrac{\sqrt{x^3}+x}{\sqrt{x}}$

$= \dfrac{x^{\left(\frac{3}{2}\right)} + x}{x^{\left(\frac{1}{2}\right)}}$

$= \dfrac{x^{\left(\frac{3}{2}\right)}}{x^{\left(\frac{1}{2}\right)}} + \dfrac{x}{x^{\left(\frac{1}{2}\right)}}$

$= x^{\left(\frac{2}{2}\right)} + x^{\left(\frac{1}{2}\right)}$

$= x + x^{\left(\frac{1}{2}\right)}$

The correct answer is D.

Practice

Simplify.

1. $\dfrac{x^{-3}}{x^3} + \sqrt[3]{x^2}$

2. $\sqrt[3]{x^3} - \dfrac{x^4}{x^4}$

3. $x^{\left(\frac{2}{5}\right)} - \sqrt[5]{x^3} \div x^{\left(\frac{1}{5}\right)} + 1$

4. $\sqrt[3]{x^2} \cdot \sqrt[2]{x^{-1}} \div \sqrt[6]{x^5} \cdot \sqrt[3]{x^{-2}}$

5. $x^{-5} + \sqrt{x^5} \div x^{\left(\frac{1}{2}\right)} - \sqrt{x^{-10}}$

6. $\sqrt[3]{x^2} + \sqrt[3]{x^3} \cdot \sqrt[3]{x}$

7. $x^{\left(\frac{1}{2}\right)} \cdot \sqrt{x^{-1}} + x^2$

8. $x^4 \cdot x^{\left(\frac{1}{4}\right)} + x^2 \cdot x^{-2}$

9. $\dfrac{1}{x^{-6}} + \sqrt{x^{-3}} \div x^{\left(-\frac{3}{2}\right)} \cdot x$

10. $\sqrt[2]{x^2} + \sqrt[3]{x^2} \cdot \sqrt[4]{x^2} \div \sqrt[5]{x^2}$

Identify the real values of x for which each of the following expressions is defined.

11. $\dfrac{3x^2}{2}$

12. $\dfrac{\sqrt{7y-9}}{x^2}$

13. $\dfrac{4k - \sqrt{65}}{6x - 2}$

14. $\dfrac{\sqrt{4y-19}}{3^{(x+2)} - 3}$

15. $\dfrac{x^3 - 6}{x^3 + x^2 + 18}$

(Answers on page 361)

ACT-TYPE PROBLEMS

1. Which of the following expressions is $\frac{4x^3y^2 + 4x^2y^3}{x + y}$ expressed in its simplest form?
   - A. $\frac{8x^5y^5}{x + y}$
   - B. $\frac{4x^2y^2(x + y)}{x + y}$
   - C. $8x^2y^2$
   - D. $4x^2y^2$
   - E. $4x^2y + 4xy^2$

2. Which of the following is $\frac{1}{\sqrt[3]{x}} \cdot \sqrt[3]{x} - \sqrt{x} \cdot \sqrt{\frac{1}{x}}$ in simplest form?
   - F. 0
   - G. 1
   - H. $1 - x$
   - J. $x^{\left(\frac{1}{3}\right)} - x^{\left(\frac{1}{2}\right)}$
   - K. $x^{\left(\frac{2}{3}\right)} - 1$

3. For which real values of x is the expression $\frac{7x}{2^{(3x-1)}}$ defined?
   - A. All real values
   - B. All real values except $\frac{1}{3}$
   - C. All real values except 0
   - D. All real values except 2
   - E. All real values except 3

4. What are the real numbers x such that $\frac{2x^2 + \sqrt{x}}{x^2 + x - 6}$ is defined?
   - F. All real numbers
   - G. All real numbers except 2
   - H. All real numbers except -3
   - J. All non-negative real numbers except 2
   - K. All non-negative real numbers except 3

5. Which of the following is $\frac{-2x^2 + 2y^2}{x - y}$ in simplest form? $(x - y \neq 0)$
   - A. $-2x + 2y$
   - B. $-2x - 2y$
   - C. $2x - 2y$
   - D. $x + y$
   - E. $x - y$

(Answers on page 361)

Solving Quadratic Equations

Quadratic equations can be written in this standard form: $ax^2 + bx + c = 0$ $(a \neq 0)$. Since the largest exponent of x is 2, the equation can have at most two solutions (roots).

It may be difficult to solve a quadratic equation by factoring.

You can always use the quadratic formula to solve a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The a, b, and c in the formula are the same as the coefficients a, b, and c in the quadratic equation.

A polynomial equation cannot have more roots than the value of its largest exponent.

EXAMPLE

Solve. $4x^2 - 2 = 3x$

Write the equation in standard form. $4x^2 - 3x - 2 = 0$

Identify the values for a, b, and c. $a = 4, b = -3, c = -2$

Substitute the values of a, b, and c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2(4)}$$

$$= \frac{3 \pm \sqrt{9 + 32}}{8}$$

$$= \frac{3 \pm \sqrt{41}}{8}$$

The solutions of the quadratic equation are $\frac{3 + \sqrt{41}}{8}$ and $\frac{3 - \sqrt{41}}{8}$.

Write the equation in standard form to identify a, b, and c.

CALCULATOR TIP

If an exact answer is not required and you have an advanced calculator, you don't need to use the quadratic formula. Just use your calculator's method for finding the roots of a polynomial.

MODEL ACT PROBLEMS

1. What is the sum of a, b, and c in the quadratic equation $23x^2 = -13x + 6$?
   - A. -4
   - B. 4
   - C. 16
   - D. 30
   - E. 42

SOLUTION

Write the equation in standard form.	$23x^2 = -13x + 6$
Add $13x$ to each side.	$23x^2 + 13x = 6$
Subtract 6 from each side.	$23x^2 + 13x - 6 = 0$
Identify the values for a, b, and c.	$a = 23, b = 13, c = -6$
Find the sum of a, b, and c.	$23 + 13 + (-6) = 36 - 6 = 30$
The correct answer is D.	

2. What are the solutions to the quadratic equation $x^2 - 2x - 35 = 0$?

F. 5 and 7
G. 5 and −7
H. 2 and 12
J. −2 and 12
K. −5 and 7

SOLUTION

Identify the values for a, b, and c.

$a = 1, b = -2, c = -35$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values of a, b, and c into the quadratic formula.

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-35)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 140}}{2}$$

$$= \frac{2 \pm \sqrt{144}}{2}$$

$$= \frac{2 \pm 12}{2} = \frac{2 + 12}{2} \text{ or } \frac{2 - 12}{2}$$

$$= 7 \text{ or } -5$$

The solutions are −5 and 7.

The correct answer is K.

Practice

1. Write the quadratic formula.
2. Identify a, b, and c in the equation $3 - 2x^2 = -8x$.

Solve each equation.

3. $x^2 - 3x - 4 = 0$
4. $5x^2 - 2 = 4x$
5. $5x + 2 = -2x^2$
6. $x^2 = x + 6$
7. $3x - 4 = -x^2$
8. $2x^2 - 5x - 3 = 0$
9. $x^2 + x = 56$
10. $x^2 - 18 = 7x$
11. $4x^2 + 9x + 2 = 0$
12. $2x^2 + 4 = -6x$
13. $x^2 - 5x = -6$
14. $4x^2 - 4x - 15 = 0$
15. $26x = -3x^2 + 9$
16. $4x^2 + 25x - 21 = 0$
17. $6x^2 = 9x + 15$
18. Mr. Wilson's rectangular garden has an area of 27 square feet. If the length of his garden is three times the width, what are the dimensions of the garden?
19. The area of a rectangular pool is 180 square feet. If the pool is 3 feet longer than it is wide, what are the dimensions of the pool?
20. Chan is framing an 8-inch by 10-inch picture. The area of the picture and the frame is 143 square inches. What is the width of the frame?

(Answers on page 362)

ACT-TYPE PROBLEMS

1. What is the product of a, b, and c in the quadratic equation $4x^2 + 2 = -14x$?

A. −112
B. −56
C. −28
D. 56
E. 112

2. What are the solutions to the quadratic equation $8x^2 = -44x - 56$?

F. −4.5 and 3
G. −3.5 and 1
H. −3.5 and −2
J. −3 and 4.5
K. −2 and 3.5

3. What are the solutions to the quadratic equation $2x^2 + 16x + 24 = 0$?

A. 2 and 6
B. 1 and 12
C. −6 and −2
D. −12 and 1
E. −12 and −1

4. What is the sum of the solutions to the quadratic equation $21x^2 - 189 = 0$?

F. −3
G. −1
H. 0
J. 1
K. 3

5. What is the product of the solutions to the quadratic equation $-29x - 22 = -6x^2$?

A. $-\frac{11}{3}$
B. $-\frac{3}{11}$
C. $\frac{4}{33}$
D. $\frac{3}{11}$
E. $\frac{33}{4}$

(Answers on page 362)

Solving Quadratic Inequalities

Factor or use the quadratic formula to solve quadratic inequalities. Use the same techniques as for quadratic equations. However, you must consider the following cases:

- If the quadratic inequality is written *less than* zero, then the two factors have different signs.

EXAMPLE

Solve. $x^2 - 9 < 0$

$(x + 3)(x - 3) < 0$

$(x + 3)$ and $(x - 3)$ must have different signs. That happens when:

$x + 3 < 0$ and $x - 3 > 0$ OR $x + 3 > 0$ and $x - 3 < 0$

$x < -3$ and $x > 3$ | $x > -3$ and $x < 3$

This is impossible. | $-3 < x < 3$

The solution is all real numbers between −3 and 3.

- If the quadratic inequality is written *greater than* zero then have the same sign.

EXAMPLE

Solve.

$x^2 - 9 > 0$

$(x + 3)(x - 3) > 0$

$(x + 3)$ and $(x - 3)$ must have the same sign. That happens when:

$x + 3 > 0$ and $x - 3 > 0$	OR	$x + 3 < 0$ and $x - 3 < 0$
$x > -3$ and $x > 3$		$x < -3$ and $x < 3$
This means the number must be greater than 3.		This means the number must be less than −3.
$x > 3$		$x < -3$

The solution is all real numbers greater than 3 or less than −3.

MODEL ACT PROBLEM

Which is the solution set of $x^2 + 2x - 8 \geq 0$?

A. $x \leq 4$
B. $x \geq 2$
C. $x \leq -4$ or $x \geq 2$
D. $-4 \leq x \leq 2$
E. $-2 \leq x \leq 4$

SOLUTION

$x^2 + 2x - 8 \geq 0$

$(x + 4)(x - 2) \geq 0$

Both factors must have the same sign.

$x + 4 \geq 0$ and $x - 2 \geq 0$	OR	$x + 4 \leq 0$ and $x - 2 \leq 0$
$x \geq -4$ and $x \geq 2$		$x \leq -4$ and $x \leq 2$
$x \geq 2$		$x \leq -4$

The correct answer is C.

Practice

Write the solution set for the given inequality.

1. $x^2 - 16 < 0$
2. $x^2 + 7x \leq 0$
3. $x^2 - 25 > 0$
4. $x^2 - 4x \geq 0$
5. $3x^2 + 10x \leq 8$
6. $4x^2 - 9 \geq 0$
7. $2x^2 - 11x + 5 \geq 0$
8. $x^2 > 8x + 20$
9. $x^2 + 27 < 12x$
10. $x^2 + 2x < 15$
11. $x^2 - 8 > 8$
12. $2x^2 - x - 3 > 0$
13. $x^2 + 1 < 2x$
14. $3x^2 - 12 \leq 0$
15. $x^2 \leq 6x - 5$
16. $16x^2 - 32 < 0$
17. $0 > -x^2 + 16$
18. $4x^2 + 5x \geq 0$
19. $x^2 + 5x \leq -4x$
20. $1{,}000{,}000x^2 \geq 0$

(Answers on page 363)

ACT-TYPE PROBLEMS

1. Which is the solution set of $x^2 - 8x + 12 < 0$?

A. $x < 2$ or $x > 6$
B. $2 < x < 6$
C. $x < -6$ or $x > -2$
D. $-6 < x < -2$
E. $-6 < x < 2$

2. Which quadratic inequality has the solution $-\sqrt{3} < x < \sqrt{3}$?

F. $x^2 - 3 < 0$
G. $x^2 - 3 > 0$
H. $x^2 - 3 \leq 0$
J. $x^2 - 3 \geq 0$
K. $x^2 + 3 < 0$

3. Which is the solution set of $x^2 + 3x - 4 > 0$?

A. $-1 < x < 4$
B. $-4 < x < 1$
C. $x < -1$ or $x > 4$
D. $x < -4$ or $x > 1$
E. $x < -4$ or $x > -1$

4. Which is the solution set of $x^2 - 3x \leq 0$?

F. $x < 0$ or $x > 3$
G. $0 < x < 3$
H. $0 \leq x \leq 3$
J. $-3 < x < 0$
K. $-3 \leq x \leq 0$

5. Which quadratic inequality has the solution set $-3 \leq x \leq 8$?

A. $x^2 + 5x - 24 < 0$
B. $x^2 - 5x + 24 \leq 0$
C. $x^2 - 5x - 24 \geq 0$
D. $x^2 - 5x - 24 < 0$
E. $x^2 - 5x - 24 \leq 0$

(Answers on page 363)

Complex Numbers

CALCULATOR TIP

Use a calculator that can represent complex numbers to check your work.

You will most frequently encounter complex numbers as you solve quadratic equations.

Imaginary Numbers

We call $\sqrt{-1}$ an imaginary number. It is neither a whole number, a decimal, nor a rational number. We use the symbol i to represent this imaginary number.

$\sqrt{-1} = i$

The Square of *i*

The square of i is −1.

$i^2 = -1$

$24i^2 = -24$

Standard Form

Every complex number has a standard form, $a + bi$, where a and b are real numbers.

Complex Addition

Treat i as a variable when you add complex numbers. Look at these examples.

$(a + bi) + (c + di) = (a + c) + (b + d)i$

$(3 + 4i) + (5 + 6i) = (3 + 5) + (4 + 6)i = 8 + 10i$

Complex Multiplication

Treat i as a variable when you multiply complex numbers, but remember that $i^2 = -1$. Look at these examples.

$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$

$(3 + 4i)(5 + 6i) = 15 + 18i + 20i + 24i^2 = (15 - 24) + (18 + 20)i = -9 + 38i$

Complex Division

The expression $(3 + 2i) \div (1 + i)$ can be written as $\frac{3 - 2i}{1 + i}$. However, this is not a complex number in the form $a + bi$. We can simplify the fraction by multiplying by the *complex conjugate* of the denominator. The **complex conjugate** of $a + bi$ is $a - bi$. Look at these examples.

$$\frac{3 - 2i}{1 + i} = \frac{3 - 2i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{3 - 3i - 2i + 2i^2}{1 - i + i - i^2} = \frac{3 - 5i - 2}{1 + 1} = \frac{1 - 5i}{2} = \frac{1}{2} - \frac{5}{2}i$$

$$\frac{3 - 4i}{2 - 6i} = \frac{3 - 4i}{2 - 6i} \cdot \frac{2 + 6i}{2 + 6i} = \frac{6 + 18i - 8i - 24i^2}{4 + 12i - 12i - 36i^2} = \frac{6 + 10i - 24}{4 + 36}$$

$$= \frac{30 + 10i}{40} = \frac{3}{4} + \frac{1}{4}i$$

Write the value of the number. Use standard form if possible.

MODEL ACT PROBLEMS

1. Which of the following choices represents $3 + 4i - 7 + 5i$ in standard form?

   A. $7i - 2i$
   B. $-4 + 9i$
   C. $-4 + 9i^2$
   D. $5i$
   E. $4 - i$

SOLUTION

Standard form for a complex number is $a + bi$. Treat i as a variable when you add complex numbers.

$3 + 4i - 7 + 5i = (3 - 7) + (4i + 5i)$

$= -4 + 9i$

The correct answer is B.

2. In simplest form, $(6 - 4i)(5 + 2i) = ?$

   F. $30 - 8i^2$
   G. $30 - 8i - 8i^2$
   H. $22 - 8i$
   J. $38 - 8i$
   K. $38 + 8i$

SOLUTION

Use FOIL. $(6)(5) + (6)(2i) - (4i)(5) - (4i)(2i)$

Simplify. $30 + 12i - 20i - 8i^2$

Combine like terms. $(30 - 8i^2) + (12i - 20i)$

Remember $i^2 = -1$. $38 - 8i$

The correct answer is J.

Practice

1. $18i^2$ 2. i^3 3. $\sqrt{-49}$ 4. $\sqrt{-50}$ 5. $\sqrt{-48}$

Write in standard form.

6. $18 - 6 + 5i$ 7. $3 \times 9 + 3i$ 8. $4i - 16 - 5i$ 9. $2 + 7i - 6 + 5i$ 10. $7(6 - \sqrt{-1})$

Add.

11. $(2 + 9i) + (3 + 7i)$ 12. $(12 + 2i) + (7 + 3i)$ 13. $(13 + 9i) + (-21 + 7i)$

14. $(12.5i^2 + 3i) + (-8 + 6i)$ 15. $\left(\frac{1}{2} + \frac{3}{4}i\right) + \left(\frac{1}{4} + \frac{1}{8}i\right)$

Multiply or divide. Express answers in $a + bi$ form.

16. $(2 + 9i)(3 + 7i)$ 17. $(12 + 2i)(7 + 3i)$ 18. $(13 + 9i)(-4 + i)$

19. $(0.5 + 3i)(-8 + 6i)$ 20. $\left(\frac{1}{2} + \frac{3}{4}i\right) \times \left(\frac{1}{4} + \frac{2}{3}i\right)$ 21. $4 \div i$

22. $\frac{5}{-2 + 6i}$ 23. $(2 - 0.5i) \div (1 + 0.5i)$ 24. $\frac{-7 + 2i}{9 + 5i}$

(Answers on page 363)

ACT-TYPE PROBLEMS

1. In standard form, $-3(6 - \sqrt{-25}) = ?$

   A. $-18 + 15i$
   B. $-18 - 15i$
   C. $-18 - 5i$
   D. -3
   E. $-3i$

2. Find the sum of $5 - 2i$ and $-3 + 7i$.

   F. $8 - 9i$
   G. $-2 + 9i$
   H. $2 - 5i$
   J. $2 + 5i$
   K. $2 + 5i^2$

3. Use the quadratic formula to find the roots of $x^2 + 4 = 0$.

   A. -2 only
   B. -2 and 2
   C. $4 + 2i$ and $4 - 2i$
   D. $-8i$ and $8i$
   E. $-2i$ and $2i$

4. Find the quadratic equation whose roots are $3i$ and $-3i$.

   F. $x^2 - 9 = 0$
   G. $x^2 + 9 = 0$
   H. $x^2 - 6ix + 9 = 0$
   J. $x^2 - 6ix - 9 = 0$
   K. $x^2 + 6ix - 9 = 0$

5. Use the quadratic formula to find the roots of $x^2 - 6x + 10 = 0$.

   A. 2 and 4
   B. 2 and 5
   C. $-3i$ and $3i$
   D. $3 + i$ and $3 - i$
   E. $3 + \sqrt{19}$ and $3 - \sqrt{19}$

(Answers on page 363)

Practice

Write the next three terms in each sequence.

1. 5, 5, 10, 15, 25, 40, 65, …
2. 1, 3, 5, 7, 9, …
3. 97, 86, 75, 64, 53, …
4. 1, 3, 4, 7, 11, 18, 29, 47, …
5. 3, 8, 13, 18, 23, …
6. 1, 8, 27, 64, 125, …
7. 2,000; 1,000; 500; 250; 125; …
8. 2, 6, 18, 54, 162, 486, …
9. 39, 31, 23, 15, 7, …
10. 2, 3, 5, 7, 11, 13, 17, 19, …
11. 2, 4, 6, 8, 10, …
12. 1, 2, 4, 8, 16, 32, …
13. 3, 6, 12, 24, 48, …
14. 1, 4, 9, 16, 25, …
15. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$
16. 15, 11, 7, 3, −1, …
17. 2, 3, 5, 7, 11, …
18. $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, \ldots$
19. 1, 3, 7, 15, 31, …
20. $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, 7, \ldots$

(Answers on page 364)

ACT-TYPE PROBLEMS

1. Gary swims every week. The table shows the total amount of time Gary swims in five consecutive weeks. If this pattern continues, how long will Gary swim in the 6th week?

Week 1	75 minutes
Week 2	84 minutes
Week 3	93 minutes
Week 4	102 minutes
Week 5	111 minutes

A. 122 minutes
B. 120 minutes
C. 118 minutes
D. 116 minutes
E. 114 minutes

2. The first term in a geometric sequence is 3, and the common factor is 2. Which of the following shows the first 5 terms in the sequence?

F. 3, 5, 7, 8, 9
G. 3, 6, 12, 24, 48
H. 2, 5, 8, 11, 14
J. 2, 6, 18, 54, 162
K. 3; 18; 108; 648; 3,888

3. Which of the following choices displays the seventh term in the sequence below?

4 32 256 2,048

A. 131,072
B. 442,368
C. 838,860
D. 1,048,576
E. 8,388,608

4. 81 is the ninth term in which of the following sequences?

F. 1, 3, 6, 10, 15, …
G. 11, 21, 31, 41, 51, …
H. 1, 4, 9, 16, 25, …
J. 8, 16, 24, 32, 40, …
K. 5, 10, 15, 20, 25, …

5. What are the next three numbers in the sequence 1, 2, 4, 8, 16, … ?

A. 24, 36, 52
B. 32, 64, 128
C. 48, 96, 192
D. 32, 48, 64
E. 48, 64, 136

(Answers on page 364)

Matrices

Matrix

A matrix is a rectangular array of numbers or variables. The entries in a matrix are called elements. Examples of matrices are shown below.

A. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ B. $\begin{bmatrix} z & y \\ x & w \\ a & b \end{bmatrix}$ C. $\begin{bmatrix} 10 & 3 & 5 & 9 \end{bmatrix}$ D. $\begin{bmatrix} -6 & 5 & 123 \\ 19 & -51 & -1.8 \\ 23 & 2 & -8 \end{bmatrix}$

The horizontal entries are called rows, while the vertical entries are called columns. Notice that there are the same number of elements in each row, and the same number of elements in each column.

Dimension

The dimension of a matrix is the number of rows followed by the number of columns. Here are the dimensions of each matrix shown above:

A. 2 × 3 B. 3 × 2 C. 1 × 4 D. 3 × 3

Scalar

Scalar is just another name for a number.

Matrix Arithmetic

You are most likely to encounter matrix addition and scalar multiplication on the ACT.

Matrix Addition

You may add matrices that have the same dimension. Just add the corresponding elements of the matrices to form a new matrix. Look at this example.

$$\begin{bmatrix} 3 & 8 \\ -9 & 6 \\ 13 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -5 \\ -3 & -14 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 3+6 & 8+(-5) \\ -9+(-3) & 6+(-14) \\ 13+0 & -6+9 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ -12 & -8 \\ 13 & 3 \end{bmatrix}$$

Scalar Multiplication

Multiply each element in the matrix by a scalar (number). Look at this example.

$$7\begin{bmatrix} 3 & 8 & -9 & 6 \\ 12 & -7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 7\times 3 & 7\times 8 & 7\times(-9) & 7\times 6 \\ 7\times 12 & 7\times(-7) & 7\times 5 & 7\times 0 \end{bmatrix} = \begin{bmatrix} 21 & 56 & -63 & 42 \\ 84 & -49 & 35 & 0 \end{bmatrix}$$

Matrix Multiplication

Multiplying two matrices is more complicated than multiplying a matrix by a scalar. Look at this example.

$$\begin{bmatrix} 5 & -4 & 2 \\ 1 & 3 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 0 & 5 \\ -2 & 0 \end{bmatrix}$$

The multiplication cannot be done if the number of columns in the 1st matrix does not equal the number of rows in the 2nd.

1st matrix 2 × 3 — 2nd matrix 3 × 2 — These must match

Step 1. Find the dimension of the product. This step is easy.
Write the dimension of the first matrix: 2 × 3
Write the dimension of the second matrix: 3 × 2
The product will have the same number of rows as the first matrix and the same number of columns as the second matrix.

1st matrix	2nd matrix
2 × 3	3 × 2

Dimension of the product

The dimension of the product will be 2 × 2.

Step 2. Multiply the matrices. For each element of the product, multiply a row in the first matrix by a column in the second matrix and add the products of the elements. For example, to find the top left element, multiply the first row in the first matrix by the first column in the second. This is highlighted below:

$$\begin{bmatrix}5 & -4 & 2\\ 1 & 3 & -2\end{bmatrix} \times \begin{bmatrix}3 & -1\\ 0 & 5\\ -2 & 0\end{bmatrix} = \begin{bmatrix}5(3)-4(0)+2(-2) & ?\\ ? & ?\end{bmatrix} = \begin{bmatrix}11 & ?\\ ? & ?\end{bmatrix}$$

Repeat this process for all the elements in the product:

$$\begin{bmatrix}5 & -4 & 2\\ 1 & 3 & -2\end{bmatrix} \times \begin{bmatrix}3 & -1\\ 0 & 5\\ -2 & 0\end{bmatrix} = \begin{bmatrix}11 & 5(-1)-4(5)+2(0)\\ 1(3)+3(0)-2(-2) & 1(-1)+3(5)-2(0)\end{bmatrix} = \begin{bmatrix}11 & -25\\ 7 & 14\end{bmatrix}$$

CALCULATOR TIP

Some calculators can add, subtract, and multiply matrices. You can always use a calculator to complete or check your calculations.

Problem Solving With Matrices

You can use matrix multiplication to solve one type of problem that may appear on your test. You may see a problem like this on the ACT:

EXAMPLE

Bob and Liz purchased bags of candy, Bags A, B, and C. Bob bought six of Bag A, four of Bag B, and nine of Bag C. Liz bought four of Bag A, eight of Bag B, and five of Bag C. Bags of candy A cost $3, bags of candy B cost $6, and bags of Candy C cost $8. What was the total cost of all the candy in Bags A, B, and C that Bob and Liz bought?

Here's how to solve it using matrix multiplication.

Write a matrix for how many bags of candy each person bought.

Write a matrix for the cost of each bag.

Number of Bags

	A	B	C
Bob	6	4	9
Liz	4	8	5

Cost

A	$3
B	$6
C	$8

Multiply: $\begin{bmatrix}6 & 4 & 9\\ 4 & 8 & 5\end{bmatrix} \times \begin{bmatrix}\$3\\ \$6\\ \$8\end{bmatrix} = \begin{bmatrix}6 \times \$3 + 4 \times \$6 + 9 \times \$8\\ 4 \times \$3 + 8 \times \$6 + 5 \times \$8\end{bmatrix}$

$$= \begin{bmatrix}\$18 + \$24 + \$72\\ \$12 + \$48 + \$40\end{bmatrix} = \begin{bmatrix}\$114\\ \$100\end{bmatrix}$$

The top entry in the final matrix shows the cost of Bob's candy.

The bottom entry shows the cost of Liz's candy.

Add the entries to find the total cost.

$114 + $100 = $214

Note that you do not need matrices to solve this problem. You can just think the problem through.

Candy A: 6 + 4 = 10 bags, Candy B: 4 + 8 = 12 bags, Candy C: 9 + 5 = 14 bags.

Then multiply by the cost:

$10 \times \$3 + 12 \times \$6 + 14 \times \$8 = \$30 + \$72 + \$112 = \$214$

MODEL ACT PROBLEM

$\begin{bmatrix}8 & -8\\ 0 & -2\end{bmatrix} + \begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix} = ?$

A. $\begin{bmatrix}6 & 6\\ -2 & 0\end{bmatrix}$

B. $\begin{bmatrix}6 & -10\\ -2 & -4\end{bmatrix}$

C. $\begin{bmatrix}6 & -6\\ 2 & 4\end{bmatrix}$

D. $\begin{bmatrix}10 & -10\\ -2 & -4\end{bmatrix}$

E. $\begin{bmatrix}-10 & 10\\ -2 & 4\end{bmatrix}$

SOLUTION

Add the corresponding elements.

$\begin{bmatrix}8 & -8\\ 0 & -2\end{bmatrix} + \begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix}$

$= \begin{bmatrix}8 + (-2) & -8 + (-2)\\ 0 + (-2) & -2 + (-2)\end{bmatrix}$

$= \begin{bmatrix}6 & -10\\ -2 & -4\end{bmatrix}$

The correct answer is B.

Practice

Use the matrices shown below to answer questions 1–15.

A. $\begin{bmatrix}-1 & 1 & -4\\ 9 & -7 & 0\\ 12 & -9 & 2\end{bmatrix}$ B. $\begin{bmatrix}3 & -8\\ 6 & 12\\ 5 & 10\end{bmatrix}$ C. $\begin{bmatrix}10 & 3 & 5 & 9\\ -3 & 6 & 5 & 2\end{bmatrix}$ D. $\begin{bmatrix}-6 & 5 & 12\\ 19 & -1 & -2\\ 23 & 2 & -8\end{bmatrix}$

E. $\begin{bmatrix}5 & -9 & 12 & 2\\ -3 & 7 & -3 & 8\end{bmatrix}$ F. $\begin{bmatrix}9 & 18 & -12\\ 0 & 3 & -6\\ -2 & 8 & 9\end{bmatrix}$ G. $\begin{bmatrix}7 & 2 & -5 & 8\end{bmatrix}$ H. $\begin{bmatrix}-1 & 3 & 2\\ 9 & -1 & -8\\ -4 & 11 & -7\\ 12 & 3 & 5\end{bmatrix}$

Write the dimension of the matrix.

1. A **2.** C **3.** B **4.** G **5.** H

Add these matrices.

6. C + E **7.** A + F **8.** D + F **9.** F + A **10.** D + H

Find the scalar product.

11. $-1 \times G$ **12.** $2 \times F$ **13.** $7 \times C$ **14.** $-2 \times B$ **15.** $0.5 \times H$

Subtract the matrices.

16. $C - E$ **17.** $A - F$ **18.** $D - F$ **19.** $F - A$ **20.** $D - H$

(Answers on page 364)

ACT-TYPE PROBLEMS

1. What are the dimensions of this matrix?

$[5 \quad 2 \quad -3]$

A. 1×3
B. 3×1
C. 3×0
D. $5 \times 2 \times -3$
E. 3

2. $\begin{bmatrix} 5 & 2 & -3 \\ 4 & -7 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ 2 & -7 \\ -3 & 0 \end{bmatrix} = ?$

F. $\begin{bmatrix} 10 & 4 & -6 \\ 8 & -14 & 0 \end{bmatrix}$

G. $\begin{bmatrix} 10 & 8 \\ 4 & -14 \\ -6 & 0 \end{bmatrix}$

H. $\begin{bmatrix} 10 & 6 & -1 \\ -3 & -10 & 0 \end{bmatrix}$

J. $\begin{bmatrix} 10 & -3 \\ 6 & -10 \\ -1 & 0 \end{bmatrix}$

K. The two matrices cannot be added.

3. Multiply.

$-3\left[4 \quad -7 \quad -\frac{1}{2}\right]$

A. $\left[-12 \quad 21 \quad -\frac{3}{2}\right]$

B. $\begin{bmatrix} -12 \\ 21 \\ \frac{3}{2} \end{bmatrix}$

C. $\left[-12 \quad 21 \quad \frac{3}{2}\right]$

D. $\left[1 \quad -10 \quad -3\frac{1}{2}\right]$

E. $\left[\frac{21}{2}\right]$

4. $[2 \quad 3 \quad 4] + [-2 \quad -3 \quad -4] = ?$

F. $\begin{bmatrix} 2 & 3 & 4 \\ -2 & -3 & -4 \end{bmatrix}$

G. $[-4 \quad -6 \quad -8]$

H. $[-4 \quad -9 \quad -16]$

J. $[0 \quad 0 \quad 0]$

K. $[0]$

5. Subtract.

$\begin{bmatrix} 1 & 3 \\ 5 & 7 \\ 9 & 11 \end{bmatrix} - \begin{bmatrix} -1 & -3 \\ -5 & -7 \\ -9 & -11 \end{bmatrix}$

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 6 \\ 10 & 14 \\ 18 & 22 \end{bmatrix}$

C. $\begin{bmatrix} -2 & -6 \\ -10 & -14 \\ -18 & -22 \end{bmatrix}$

D. $\begin{bmatrix} 8 \\ 24 \\ 40 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 3 & -1 & -3 \\ 5 & 7 & -5 & -7 \\ 9 & 11 & -9 & -11 \end{bmatrix}$

6. A car dealership had a special sale on Wednesday and another on Thursday. Convertibles (C), midsized cars (M), and SUVs (S) were sold on Wednesday and Thursday. The matrices below show the number of each type of car sold for each day and the bonus the dealer received for each car sold from the manufacturer. What total bonus did the dealership receive from the manufacturer for the convertibles, midsized cars, and SUVs sold on those two sale days?

$$\begin{array}{r} \\ \text{Wednesday} \\ \text{Thursday} \end{array} \begin{array}{c} \begin{array}{ccc} C & M & S \end{array} \\ \begin{bmatrix} 24 & 38 & 13 \\ 32 & 52 & 11 \end{bmatrix} \end{array} \qquad \begin{array}{r} \\ C \\ M \\ S \end{array} \begin{array}{c} \text{Bonus} \\ \begin{bmatrix} \$200 \\ \$100 \\ \$300 \end{bmatrix} \end{array}$$

F. \$10,200
G. \$12,500
H. \$14,900
J. \$27,400
K. \$30,800

7. $[450 \quad 700 \quad 900] \times \begin{bmatrix} 28 & 81 & 41 \\ 30 & 36 & 32 \\ 51 & 25 & 29 \end{bmatrix} = ?$

A. $[79{,}500 \quad 84{,}150 \quad 66{,}950]$

B. $[49{,}050 \quad 99{,}400 \quad 91{,}800]$

C. $[230{,}600]$

D. $\begin{bmatrix} 67{,}500 \\ 68{,}600 \\ 94{,}500 \end{bmatrix}$

E. $\begin{bmatrix} 79{,}500 \\ 84{,}150 \\ 66{,}950 \end{bmatrix}$

(Answers on page 364)

8. The Amsco designer shirts were hot sellers. Amsco sells three types of T-shirts: regular T-shirts (R), T-shirts with collars (C), and long sleeve T-shirts (L). The T-shirts are sold in three stores located in New York, Chicago, and Detroit. The matrix below gives the number of shirts sold at each store on opening day.

$$\begin{array}{r} \\ \text{New York} \\ \text{Chicago} \\ \text{Detroit} \end{array} \begin{array}{c} \begin{array}{ccc} R & C & L \end{array} \\ \begin{bmatrix} 120 & 75 & 35 \\ 125 & 60 & 25 \\ 90 & 35 & 40 \end{bmatrix} \end{array}$$

The price of each T-shirt is shown in the following matrix.

$$\begin{array}{r} R \\ C \\ L \end{array} \begin{bmatrix} \$22.50 \\ \$29.00 \\ \$36.50 \end{bmatrix}$$

Given these matrices, what were the total sales for opening day?

F. \$16,117.50
G. \$20,247.50
H. \$25,975
J. \$27,264
K. \$67,145

Chapter 10

Coordinate Geometry

- Nine ACT questions have to do with coordinate geometry.
- Easier coordinate geometry questions may be about a single skill or concept, or may test a combination of pre-algebra, elementary algebra, intermediate algebra, and coordinate skills.
- More difficult questions will often test a combination of coordinate geometry skills and concepts.
- This coordinate geometry review covers all the material you need to answer ACT questions.
- Use a calculator for the ACT-Type Problems. Do not use a calculator for the Practice exercises.

Graphing Inequalities on a Number Line

You can graph an inequality on a number line.

- An open circle, ○, shows that a point is not included.
- A closed circle, ●, shows that a point is included.
- An arrow, ← or →, shows that the line continues forever in that direction.

Graphing Inequalities

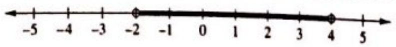
EXAMPLES

1. Graph. $x > -3$

The graph of $x > -3$ should show all points greater than -3 but not including -3. Draw an open circle at -3 and an arrow to the right. The open circle shows that -3 is not included. The arrow shows that the graph goes on forever to the right.

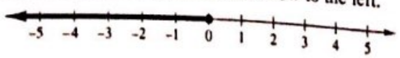
2. Graph. $-2 < x < 4$

The graph of $-2 < x < 4$ should show all points between -2 and 4 but not including -2 or 4. Draw open circles at -2 and 4. Shade the number line between the open circles.



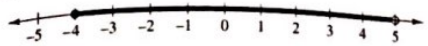
3. Graph. $x \le 0$

The graph of $x \le 0$ should show all points less than 0, including 0. Draw a closed circle at 0 and an arrow to the left.



4. Graph. $-4 \le x < 5$

The graph of $-4 \le x < 5$ should show all the points between -4 and 5, including -4 but not including 5. Draw a closed circle for -4 and an open circle for 5. Shade the number line between the circles.



CALCULATOR TIP

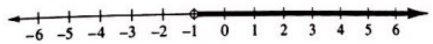
Some graphing calculators can plot inequalities on a number line. You can always use a calculator to complete or check your calculations.

5. Graph. $2x + 1 < 3x + 2$

First, solve the inequality.

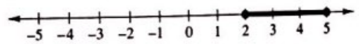
$$\begin{array}{rcl} 2x + 1 & < & 3x + 2 \\ -2 & & -2 \\ \hline 2x - 1 & < & 3x \\ -2x & & -2x \\ \hline -1 & < & x \end{array} \qquad (x > -1)$$

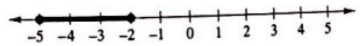
Then, graph the solution.

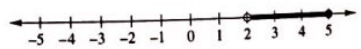


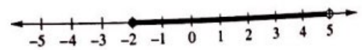
MODEL ACT PROBLEMS

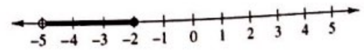
1. Which of the following is the graph of the inequality $-2 \le x < 5$?

A. 

B. 

C. 

D. 

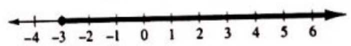
E. 

SOLUTION

The inequality $-2 \le x < 5$ represents a section of the number line from -2 to 5 with -2 included and 5 excluded.

The correct answer is D.

2. Which inequality is graphed below?



F. $x > -3$
G. $3(x + 1) \ge 3(5 - x)$
H. $x + 1 \ge -5\left(1 + \frac{x}{5}\right)$
J. $x + 1 \ge 5 + x$
K. $x + 1 \ge -x + 4$

SOLUTION

Since the circle at -3 is closed and the arrow shows that the graph goes on forever to the right, the inequality shown is $x \ge -3$.

Find the inequality with the solution $x \ge -3$.

$$x + 1 \ge -5\left(1 + \frac{x}{5}\right)$$
$$x + 1 \ge -5 - x$$
$$2x \ge -6$$
$$x \ge -3$$

The correct answer is H.

1, 2, 3, 4

Practice

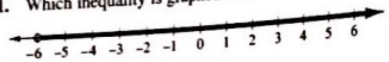
Graph each inequality on a number line.

1. $x < -1$
2. $x \geq -2$
3. $x \geq -1$
4. $x \geq 3$
5. $x < -3$
6. $x \geq 4$
7. $x > -5$
8. $5 \geq x > -2$
9. $-3 \leq x \leq 1$
10. $2 \geq x > -2$
11. $-3 < x \leq -1$
12. $2 \leq x \leq 4$
13. $-1 < x \leq 3$
14. $4 \geq x \geq 1$
15. $3x + 1 \geq 2x - 2$
16. $3x - 1 \leq 4x + 3$
17. $9x + 7 < 2x + 28$
18. $-5x - 2 \geq 3x + 14$
19. $21x + 9 > 14x + 2$
20. $x - 15 \leq 20x + 23$

(Answers on page 367)

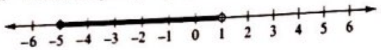
ACT-TYPE PROBLEMS

1. Which inequality is graphed below?



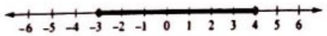
A. $x < 6$
B. $x > -6$
C. $x \leq -6$
D. $x \geq -6$
E. $x \leq 6$

2. Which inequality is graphed below?



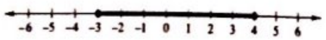
F. $x \geq -5$
G. $-5 \leq x \leq 1$
H. $x \leq 1$
J. $-5 \leq x < 1$
K. $-5 < x \leq 1$

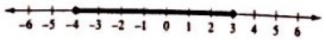
3. Which of the following is the graph of the inequality $-3 < x < 4$?

A.


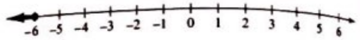
B.

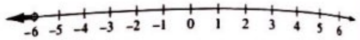

C.

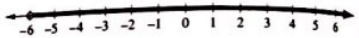

D.


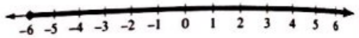
E.


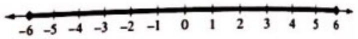
4. Which of the following is the graph of the inequality $3x - 5 \leq 5x + 7$?

F.


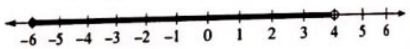
G.


H.


J.


K.


5. What is the sum of all integers that are solutions of the inequality graphed below?



A. −11
B. −12
C. −13
D. −14
E. −15

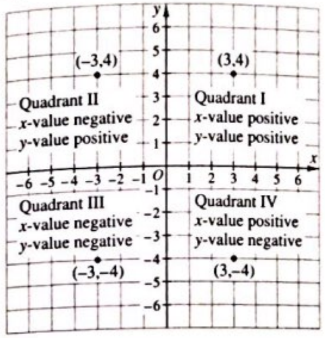
(Answers on page 367)

Graphing Equations on the Coordinate Plane

A linear equation can be written in the form $ax + by = c$.

Points on the Coordinate Plane

A point on the coordinate plane is named by an ordered pair (x,y). The x refers to the value on the x (horizontal) axis. The y refers to the value on the y (vertical) axis. Look at the points plotted below.



CALCULATOR TIP

Graphing calculators are designed to graph equations and inequalities on the coordinate plane.

EXAMPLES

1. Graph. $3x + 2y = 6$

Solve the equation for $x = 0$.

$$0 + 2y = 6$$
$$y = 3$$

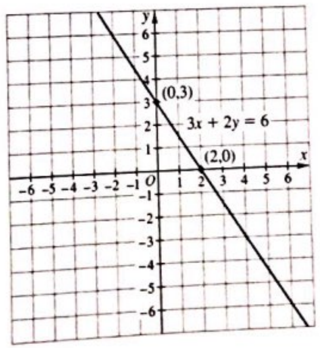
One ordered pair is (0,3).

Solve the equation for $y = 0$.

$$3x + 0 = 6$$
$$x = 2$$

A second ordered pair is (2,0).

Plot the points and connect them.



When x = 0, the solution line crosses the y-axis. When y = 0, the solution line crosses the x-axis.

2. Graph. $8x - 3y + 12 = 0$

Write the equation in standard form.

$8x - 3y = -12$

Solve the equation for $x = 0$.

$0 - 3y = -12$

$y = 4$

One ordered pair is (0,4).

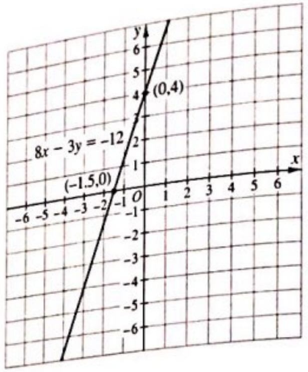
Solve the equation for $y = 0$.

$8x + 0 = -12$

$x = -1.5$

A second ordered pair is $(-1.5,0)$.

Plot the points and connect them.



Slope

Slope Formula

In the slope formula, m stands for the slope of a line. To find the slope, identify two points on the line, (x_1,y_1) and (x_2,y_2). The slope is the difference of the y-values divided by the difference of the x-values as long as the x-values are not equal. You should memorize this slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ with } x_2 - x_1 \neq 0$$

A line with a slope of 0 is parallel to the x-axis. A line with an undefined slope is parallel to the y-axis.

EXAMPLES

1. Find the slope of the line passing through the points (2,6) and (3,5).

   Label one ordered pair (x_1,y_1) and the other pair (x_2,y_2).

   $(x_1,y_1) \rightarrow (2,6)$ $\quad (x_2,y_2) \rightarrow (3,5)$

   Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of the line passing through the points.

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{3-2} = \frac{-1}{1} = -1$$

   The slope is -1.

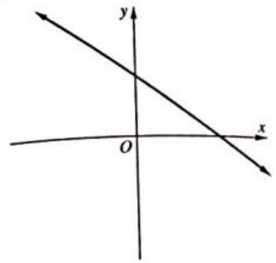
2. Find the slope of the line passing through the points $(-2,0)$ and (0,4).

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$$

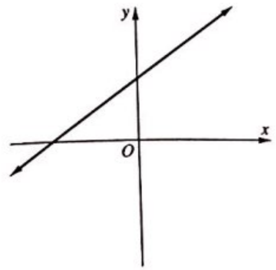
   The slope is 2.

Slopes of Lines

- Lines that slope down from left to right have a negative slope.



- Lines that slope up from left to right have a positive slope.



- Vertical lines have no slope. The slope of a vertical line is undefined. All x-values are the same.
- Horizontal lines have a slope of 0. All y-values are the same.

Slopes of Pairs of Lines

- Parallel lines have the same slope.
- Perpendicular lines have slopes whose product is -1 (except when one of the lines is vertical). The slopes of perpendicular lines are negative reciprocals.

Slope-Intercept Form

Every linear equation can be written in slope-intercept form, which is given below.

$y = mx + b$

- m is the slope.
- b is the y-intercept (where the line crosses the y-axis and $x = 0$).

You should memorize this equation.

EXAMPLE

Graph. $8x - 4y = 6$

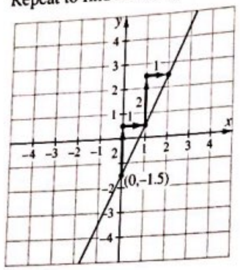
Write the equation in slope-intercept form. $8x - 4y = 6$

$-4y = -8x + 6$

Solve the equation for y. $y = 2x - 1.5$

From the equation, you can see that the slope is 2 and the y-intercept is -1.5.

To graph the equation, first plot the y-intercept, $(0,-1.5)$. Then use the slope to find another point on the line. From $(0,-1.5)$, go up 2 and to the right 1. Plot the point. Repeat to find another point. Then draw the line.



CALCULATOR TIP

Most graphing calculators can graph linear equations that are written in slope-intercept form. Just enter the slope-intercept form directly into the calculator. You do not have to find any ordered pairs.

Graphing to Find the Solution to Systems of Equations

The solution to a system of two equations is the point at which the two graphs intersect. If the two lines are parallel, there is no solution. If the two lines are the same, there are an infinite number of solutions.

CALCULATOR TIP

Use the INTERSECT function on your graphing calculator to find the exact point at which two lines intersect.

EXAMPLES

1. Solve the system by graphing. $x + y = 4$, $2x - y = 5$

Write each equation in slope-intercept form.

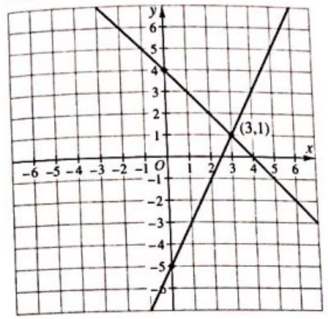
$x + y = 4$
$y = -x + 4$

The y-intercept is 4.
The slope is -1.

$2x - y = 5$
$-y = -2x + 5$
$y = 2x - 5$

The y-intercept is -5.
The slope is 2.

Graph to find the point of intersection.



The lines intersect at (3,1). So the solution to the system is $x = 3$ and $y = 1$.

2. Solve the system by graphing. $-x + y = 3$, $x - y = -6$

Write each equation in slope-intercept form.

$-x + y = 3$
$y = x + 3$

The y-intercept is 3.
The slope is 1.

$x - y = -6$
$-y = -x - 6$
$y = x + 6$

The y-intercept is 6.
The slope is 1.

Since the lines have the same slope, their graphs are parallel.

So the system has no solution.

> There is no need to actually graph the equation to find the solution since we can tell from the slopes that the lines are parallel.

3. Solve the system by graphing. $-2x - 4y = -6$, $x + 2y = 3$

Write each equation in slope-intercept form.

$-2x - 4y = -6$
$-4y = 2x - 6$
$y = -\frac{1}{2}x + \frac{3}{2}$

$x + 2y = 3$
$2y = -x + 3$
$y = -\frac{1}{2}x + \frac{3}{2}$

Since the lines have the same equation, their graphs are the same.

So the system has an infinite number of solutions.

> There is no need to actually graph the equation to find the solution since we can tell from the slopes and intercepts that the lines are the same.

MODEL ACT PROBLEMS

1. What is the slope of a line perpendicular to $y = -3x - 4$?

   A. -3
   B. -4
   C. 4
   D. $\frac{1}{4}$
   E. $\frac{1}{3}$

SOLUTION

The slopes of perpendicular lines have a product of -1.

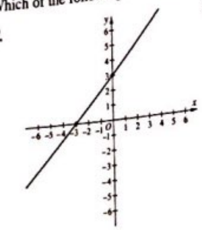
The slope of the given line is -3.

Since $-3 \times \frac{1}{3} = -1$, the slope of a perpendicular line is $\frac{1}{3}$.

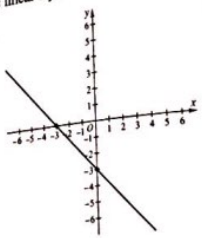
The correct answer is E.

2. Which of the following is the graph of the linear equation $x + y = -3$?

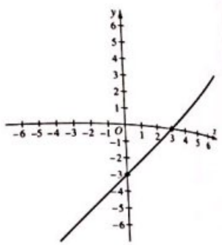
F.



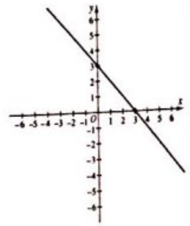
G.



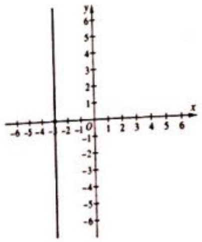
H.



J.



K.



SOLUTION

Find two points that lie on the graph of $x + y = -3$.

Let $x = 0$.	Let $y = 0$.
$0 + y = -3$	$x + 0 = -3$
$y = -3$	$x = -3$

One point is $(0,-3)$. Another point is $(-3,0)$.

The graph in G contains these points.

The correct answer is G.

3. How many solutions are there to the system of equations $y = 3x + 2$ and $12x - 4y = 8$?

   A. 0
   B. 1
   C. 1,000,000
   D. 1,000,000,000
   E. infinite

SOLUTION

Write each equation in slope-intercept form.

$$y = 3x + 2 \qquad 12x - 4y = 8$$
$$-4y = -12x + 8$$
$$y = 3x - 2$$

The equations represent different lines with the same slope.

When the slopes are the same, the lines are parallel.

Parallel lines never intersect, so there are no solutions.

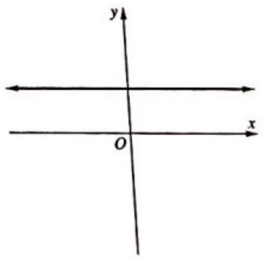
The correct answer is A.

Practice

1. Plot each point on the coordinate plane. $(-2,5)$, $(4,3)$, $(2,-3)$, $(-3,-1)$
2. What are the coordinates of the point on the y-axis and the coordinates of the point on the x-axis of the graph of $2x + 4y = 4$?
3. Graph the line with equation $3x + 6y = 12$ by plotting points.
4. What is the slope of the line whose equation is given in problem 3?
5. Find the slope and the y-intercept of the line with equation $10x + 5y = 20$.
6. Graph the line with equation $4x + 2y = 8$ using slope-intercept form.

State whether the slope of each line is positive, negative, zero, or undefined.

7.



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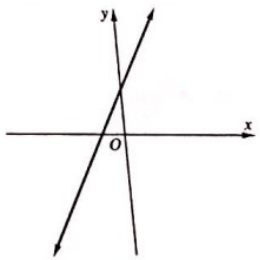

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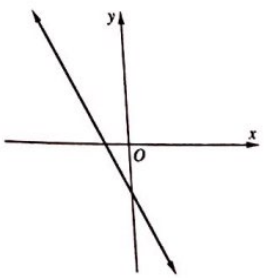

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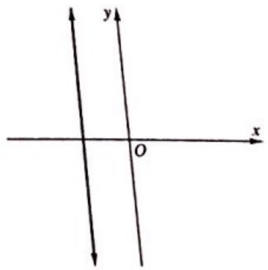


Tell whether each statement is true or false.

11. The lines represented by the following equations are parallel.

$y = -2x + 3$
$y = -3x + 3$

12. The lines represented by the following equations are perpendicular.

$y = \frac{1}{4}x + 6$

$y = -4x - 18$

13. The lines represented by the following equations are parallel.

$y = 3x - 4$
$y = 3x + 10$

14. The lines represented by the following equations are perpendicular.

$4y = x - 5$
$y = -4x + 5$

15. The y-intercept of $y = 2x - 19$ is 2.

16. There are an infinite number of solutions to the following system of equations.

$3x - 8y = 6$

$y = \frac{3}{8}x - \frac{3}{4}$

17. The slope of $2x + 4y = 20$ is 5.

Solve each problem.

18. What is the slope of the line passing through the points with coordinates (1,5) and (3,8)?

19. What is the slope of the line passing through the points with coordinates (4,7) and (7,9)?

20. Are the lines described in problems 18 and 19 parallel, perpendicular, or neither?

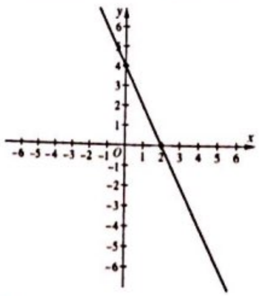
(Answers on page 368)

ACT-TYPE PROBLEMS

1. What is the sum of the y-intercept and the slope of the linear equation $6x - 9y = 3$?

A. -6
B. -3
C. $\frac{1}{3}$
D. $-\frac{1}{3}$
E. 9

2. Which equation is graphed below?



F. $y = -2x + 4$
G. $8y + 16x = 24$
H. $y = 2x - 4$
J. $2y - 4x = 8$
K. $y = 4x - 2$

3. What is the slope of a line parallel to the line with equation $12x - 3y = 17$?

A. $-\frac{17}{3}$
B. 4
C. $\frac{1}{4}$
D. -4
E. $\frac{3}{17}$

4. Which equation creates an infinite number of solutions when solved in a system with $y = 5x - 7$?

F. $2y + 10x = -14$
G. $y = 7x - 5$
H. $3y - 15x = -28$
J. $4y - 20x = -28$
K. $4y + 15x = -21$

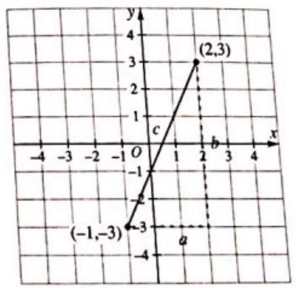
5. What is the equation of a line with a y-intercept of -3 that is perpendicular to the line with equation $3y - 4x = 21$?

A. $y = \frac{4}{3}x - 3$
B. $y = \frac{3}{4}x + 3$
C. $y = -3x - \frac{4}{3}$
D. $y = 3x + \frac{3}{4}$
E. $y = -\frac{3}{4}x - 3$

(Answers on page 368)

Distance and Midpoint Formulas

- Use the distance formula to find the distance between two points on a plane.
- Use the midpoint formula to find the midpoint of a line segment.



Finding distance is like finding the length of the hypotenuse of a right triangle. $a^2 + b^2 = c^2$

Distance Formula

Use the following formula to find the distance between two points on the plane.

The distance (d) between (x_1,y_1) and $(x_2,y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

EXAMPLE

Find the distance between the two points $(-1,-3)$ and $(2,3)$ shown in the diagram on the previous page.

$$\text{Distance} = \sqrt{(2-(-1))^2 + (3-(-3))^2}$$
$$= \sqrt{(2+1)^2 + (3+3)^2}$$
$$= \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

The distance between the two points is $3\sqrt{5} \approx 6.71$.

The symbol $\approx$ means "is approximately equal to."

Midpoint Formula

Use this formula to find the midpoint of the line segment between two points in the plane.

The midpoint of the line segment between (x_1,y_1) and $(x_2,y_2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

EXAMPLE

Find the midpoint of the line segment between the two points $(-1,-3)$ and $(2,3)$ shown in the diagram on the previous page.

$$\text{Midpoint} = \left(\frac{-1+2}{2}, \frac{-3+3}{2}\right)$$
$$= \left(\frac{1}{2}, \frac{0}{2}\right) = \left(\frac{1}{2}, 0\right)$$

The midpoint is $\left(\frac{1}{2}, 0\right)$.

This is like averaging the x-coordinates and then averaging the y-coordinates.

MODEL ACT PROBLEM

1. What is the distance between points (2,6) and (1,4)?

A. $\sqrt{29}$
B. $\sqrt{13}$
C. $\sqrt{5}$
D. 5
E. 13

SOLUTION

Identify each point. $(x_1,y_1) \rightarrow (2,6)$
$(x_2,y_2) \rightarrow (1,4)$

Use the distance formula.

$$\text{Distance} = \sqrt{(1-2)^2 + (4-6)^2}$$
$$= \sqrt{(-1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

The correct answer is C.

2. What is the midpoint of the line segment between the points (3,6) and (2,4)?

F. (2.5,5)
G. (4.5,3)
H. (3.5,4)
J. (0.5,1)
K. (1.5,1)

SOLUTION

Identify the points. $(x_1,y_1) \rightarrow (3,6)$
$(x_2,y_2) \rightarrow (2,4)$

Use the midpoint formula.

$$\text{Midpoint} = \left(\frac{3+2}{2}, \frac{6+4}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{10}{2}\right) = (2.5,5)$$

The correct answer is F.

CALCULATOR TIP

Calculators do not usually have special functions for finding the midpoint and distance. However, you can still use a calculator to check your answer.

Practice

1. What is the general formula for the distance between two points (x_1,y_1) and (x_2,y_2)?
2. What is the distance between points (3,5) and (1,2)?
3. What is the distance between points (−2,7) and (4,−6)?
4. What is the general midpoint formula given two points (x_1,y_1) and (x_2,y_2)?
5. What is the midpoint of the line segment between points (4,2) and (6,4)?
6. What is the midpoint of the line segment between points (9,1) and (2,6)?
7. What is the distance between points (5,−2) and (1,3)?
8. What is the distance between points (4,−6) and (−5,2)?
9. What is the distance between points (6,3) and (8,9)?
10. What is the distance between points (0,0) and (−3,4)?
11. What is the distance between points (−3,7) and (9,5)?
12. What is the distance between points (1,8) and (7,4)?
13. What is the distance between points (−3,9) and (−7,2)?
14. What is the midpoint of the line segment between points (5,−2) and (1,3)?
15. What is the midpoint of the line segment between points (4,−6) and (−5,2)?
16. What is the midpoint of the line segment between points (6,3) and (8,9)?
17. What is the midpoint of the line segment between points (0,0) and (−3,4)?
18. What is the midpoint of the line segment between points (−3,7) and (9,5)?
19. What is the midpoint of the line segment between points (1,8) and (7,4)?
20. What is the midpoint of the line segment between points (−3,9) and (−7,2)?

(Answers on page 369)

ACT-TYPE PROBLEMS

1. What is the distance between points $(-4,7)$ and $(5,-3)$?

   A. 19
   B. $\sqrt{181}$
   C. 9
   D. $\sqrt{81}$
   E. 3

2. What is the midpoint of the line segment between points (2,6) and (3,8)?

   F. (2,2.5)
   G. (2.5,2)
   H. (5,4.5)
   J. (4,5.5)
   K. (2.5,7)

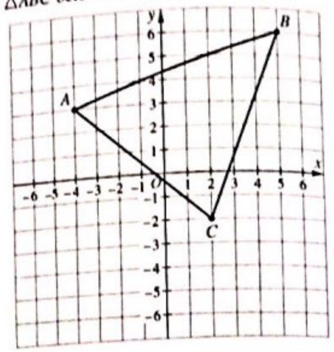
3. What is the sum of the length of the line segment between the points $(-1,5)$ and (3,8), and the y-coordinate of the midpoint of that line segment?

   A. 11.5
   B. 12
   C. 12.5
   D. 13
   E. 13.5

4. The midpoint of a line segment is (3,4). One of the endpoints of that line segment is (7,4). What is the other endpoint?

   F. (1,5)
   G. $(-4,0)$
   H. $(-1,4)$
   J. $(0,-4)$
   K. $(4,-1)$

5. Which of the following lists the length of each side of $\triangle ABC$ below from shortest to longest?



   A. *BC*, *AC*, *AB*
   B. *AC*, *AB*, *BC*
   C. *BC*, *AB*, *AC*
   D. *AC*, *BC*, *AB*
   E. *AB*, *AC*, *BC*

(Answers on page 369)

Graphing Systems of Inequalities on the Coordinate Plane

An inequality is shown as a region on the coordinate plane.

- Inequalities containing ≤ or ≥ have a solid-line boundary.
- Inequalities containing < or > have a dotted-line boundary.

Graphing an Inequality on the Coordinate Plane

To graph an inequality on the coordinate plane:

- Graph the related equation to show the boundary line.
- Pick a point on one side of the boundary line. If the point satisfies the inequality, shade the region. If the point does NOT satisfy the inequality, shade the region on the other side of the line.

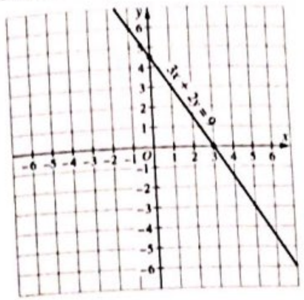
EXAMPLE

Graph. $3x + 2y \leq 9$

Graph the related equation $3x + 2y = 9$.

Let $x = 0$.	Let $y = 0$.
$3(0) + 2y = 9$	$3x + 2(0) = 9$
$y = 4.5$	$x = 3$
The ordered pair is (0,4.5).	The ordered pair is (3,0).

Plot the two points and draw the line.

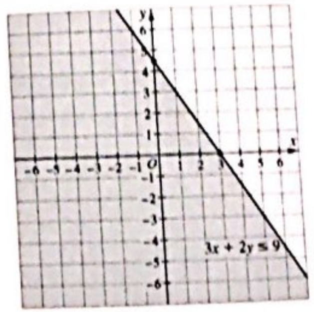


Since the inequality is ≤, the boundary is a solid line.

To find the region to shade, test (0,0) in the inequality.

$$\begin{aligned} 3x + 2y &\leq 9 \\ 3(0) + 2(0) &\leq 9 \\ 0 &\leq 9 \quad \text{True} \end{aligned}$$

Shade the region that includes the point (0,0). The shaded region and the boundary line make up the graph of the inequality $3x + 2y \leq 9$.



(0,0) is an easy point to test when it is not a point on the line.

NUMBE
44–48
1, 2, 3, 4
Ray

Graphing Two Inequalities on the Coordinate Plane

To graph two inequalities on the coordinate plane:

- Graph each inequality on the same coordinate plane.
- The portion of the shaded region common to both inequalities is the graph of the two inequalities.

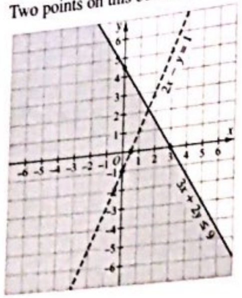
EXAMPLE

Graph. $3x + 2y \le 9$ and $2x - y > 1$

The graph of $3x + 2y \le 9$ is shown on the previous page.

To graph $2x - y > 1$, begin by graphing the equation $2x - y = 1$.

Two points on this boundary line are (0,−1) and (0.5,0).



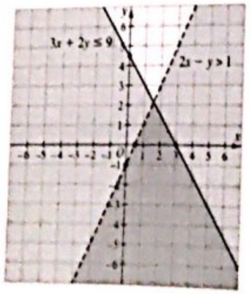
Since the inequality is >, the boundary is a dotted line. The same is true for <.

To find out which side of the boundary line to shade, test (0,0) in the inequality $2x - y > 1$.

$$2x - y > 1$$
$$2(0) - 0 > 1$$
$$0 > 1 \quad \text{False}$$

The graph of $2x - y > 1$ will *not* include the point (0,0). Shade the region to the right of the boundary line.

The shaded region that is common to both inequalities is the graph of $3x + 2y \le 9$ and $2x - y > 1$.



MODEL ACT PROBLEMS

1. Consider the following inequalities.

I. $2x + 4y > 6$
II. $3x - 2y \ge 7$
III. $5x + y < 8$
IV. $7x + 9y \le 3$

Which choice lists all the inequalities that have graphs with solid boundary lines?

A. I and III
B. I and II
C. III and IV
D. II and IV
E. II and III

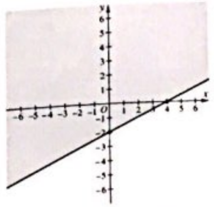
SOLUTION

II and IV have ≥ or ≤, which result in a solid boundary line.

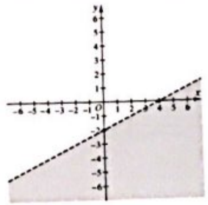
The correct answer is D.

2. Which of the following choices shows the graph of the inequality $2x - 4y < 8$?

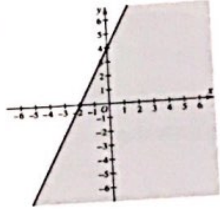
F.



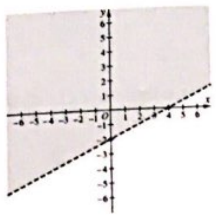
G.



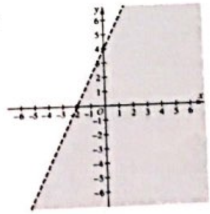
H.



J.

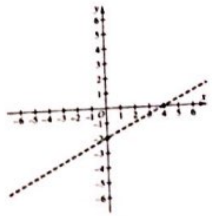


K.



SOLUTION

Graph the line $2x - 4y = 8$ as a dotted boundary line because the inequality is <.



Shade the side of the boundary line that satisfies the inequality. The graph in J shows the correct shading.

The correct answer is J.

CALCULATOR TIP

Some graphing calculators can plot inequalities. You can, however, use the calculator to graph the equations to find the boundary lines. Then test points to identify mentally the region of the graph that should be shaded in.

Practice

Should a solid line or a dotted line be used to graph each inequality?

1. $6x - 3y < 2$
2. $2x + 7y \geq 5$
3. $5x - 9y \leq 15$
4. $4x + 13y < 2$
5. When graphing two inequalities simultaneously, which part of the graph do you shade?

Graph each inequality.

6. $2x + y \geq 4$
7. $3y - 9x < 18$
8. $-3x - 6y < 12$
9. $-18x + 2y \leq 9$
10. $2x - y \geq -6$
11. $8x - 2y > 12$
12. $7x + 4y \leq -14$
13. $3x + 8y \geq 18$

Graph each system of inequalities.

14. $4x + 2y < 8$ and $-3x + y \geq -5$
15. $2x + 2y \geq 3$ and $-6x + 3y < 9$
16. $x + y < 6$ and $x - 2y \leq -3$
17. $-3x + 6y \geq -18$ and $7x - 7y < -7$
18. $2x - 5y > 10$ and $6x - 2y \geq 6$
19. $-8x - 4y < -16$ and $4x - 2y \leq 8$
20. $-4x - y < -2$ and $-4.5x - 3y \leq -9$

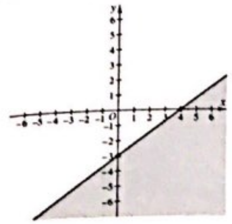
(Answers on page 370)

ACT-TYPE PROBLEMS

1. Which is NOT a solution of the inequality $2y - 6x \leq 13$?

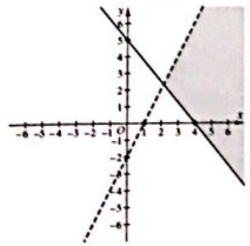
A. (0,0)
B. (2,4)
C. (1,6)
D. (2,13)
E. (2,10)

2. Which inequality is graphed below?



F. $4x - 3y \leq 12$
G. $3x + 4y \geq 12$
H. $3x - 4y \geq 12$
J. $4x - 3y < 12$
K. $3x - 4y > 12$

3. Which system of inequalities is graphed below?



A. $4y + 5x \geq 20$
$2y - 4x < -4$

B. $5y - 4x \geq 20$
$4y - 2x < -2$

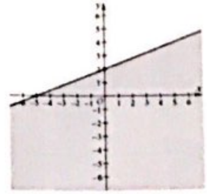
C. $4y - 5x < 20$
$2y - 4x \leq -4$

D. $5y - 4x > 20$
$4y - 2x \leq -2$

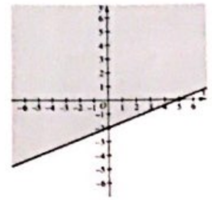
E. $-4y + 5x \leq 20$
$-2y + 4x < -4$

4. Which is the graph of the inequality $-2x + 5y \leq 10$?

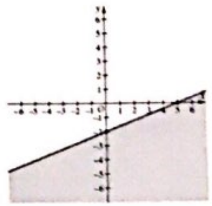
F.



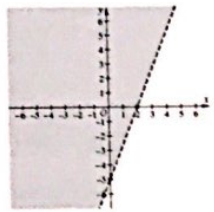
G.



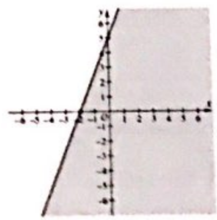
H.



J.

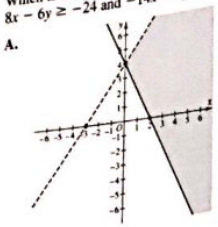


K.

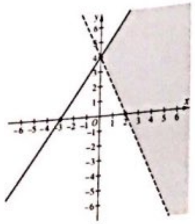


5. Which is the graph of the system of inequalities $8x - 6y \geq -24$ and $-14x - 7y < -28$?

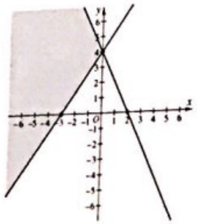
A.



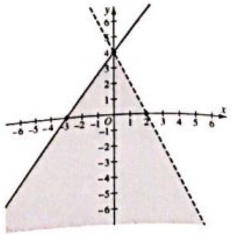
B.



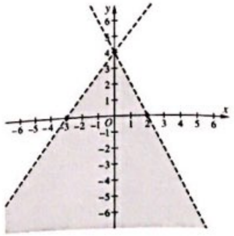
C.



D.



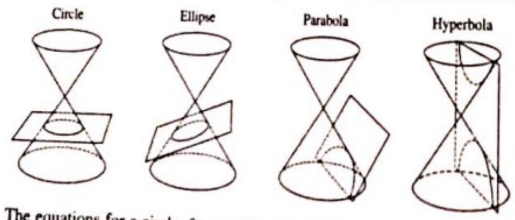
E.



(Answers on page 372)

Graphing Conic Sections

Conic sections are the figures formed by the intersection of a cone or cones and a plane.



The equations for a circle, for an ellipse, for a parabola, and for a hyperbola each have a standard form.

CALCULATOR TIP

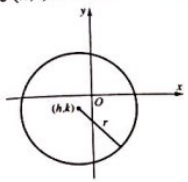
A graphing calculator can graph conic sections. Be careful when you enter the equation for circles, ellipses, and hyperbolas. These figures have two y-values for each x-value and you may have to use additional keys to display the entire graph.

Circle

The standard form of the equation for a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h,k) is the center and r is the radius.



You may have to simplify or rearrange terms to write an equation in standard form.

EXAMPLE

Graph. $x^2 + y^2 - 6x - 10y + 25 = 0$

To determine if this is an equation of a circle, first write the equation in standard form.

$$x^2 + y^2 - 6x - 10y + 25 = 0$$

Rearrange the terms. $(x^2 - 6x) + (y^2 - 10y) = -25$

To make each expression a perfect square, add the square of half the coefficient of each variable to both sides of the equation. This is called *completing the square*.

$$x^2 - 6x + \left(\frac{6}{2}\right)^2 + y^2 - 10y + \left(\frac{10}{2}\right)^2 = -25 + \left(\frac{6}{2}\right)^2 + \left(\frac{10}{2}\right)^2$$

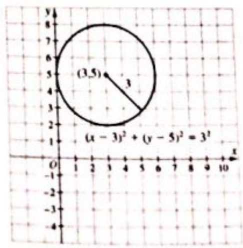
$$x^2 - 6x + 9 + y^2 - 10y + 25 = -25 + 9 + 25$$

Write each expression as a perfect square.

$$(x - 3)^2 + (y - 5)^2 = 9$$

This is the equation of a circle with center (3,5) and radius 3.

Graph the equation.



Ellipse

An ellipse is all points such that the sum of the distances from any point to two fixed points called **foci** is constant. The standard form of the equation for an ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

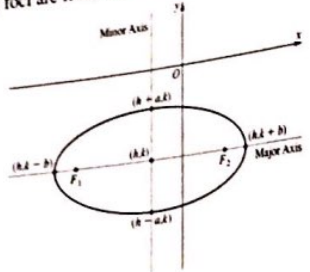
where (h,k) is the center, a and b are half the lengths of the axes of the ellipse, and the ellipse crosses the axes at $(h + a,k)$, $(h - a,k)$, $(h,k - b)$, and $(h,k + b)$.

If a^2 is the larger denominator, then the major (longer) axis is horizontal.

If b^2 is the larger denominator, then the major (longer) axis is vertical.

The foci are located on the major axis, $\sqrt{a^2 - b^2}$ units from the center.

An ellipse looks like a flattened circle.



EXAMPLE

Graph. $\frac{(x+3)^2}{36} + \frac{(y-2)^2}{4} + 6 = 7$

First write the equation in standard form.

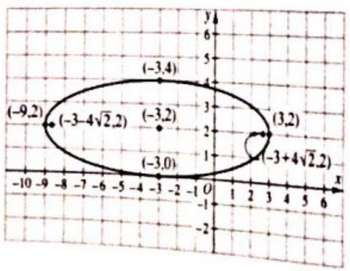
$$\frac{(x-(-3))^2}{6^2} + \frac{(y-2)^2}{2^2} = 1$$

The center is $(-3,2)$. a^2 is larger than b^2, so the major axis is horizontal.

The ellipse crosses its axes at $(3,2)$, $(-9,2)$, $(-3,0)$, and $(-3,4)$.

The foci of the ellipse are $\pm\sqrt{36-4} = \pm\sqrt{32} = \pm 4\sqrt{2}$ units along the major axis from the center, or at the points $(-3 + 4\sqrt{2},2)$ and $(-3 - 4\sqrt{2},2)$.

Use this information to graph the ellipse.

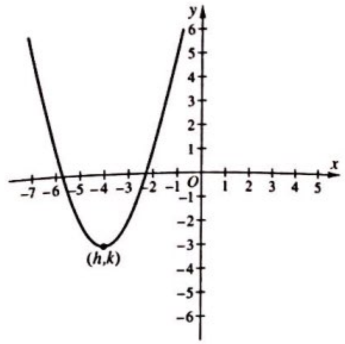


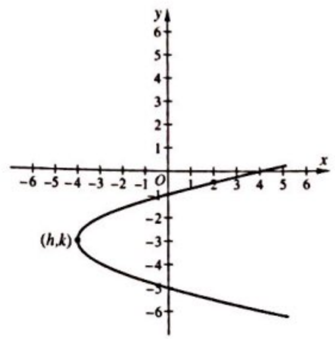
Parabola

There are two standard forms for a parabola.

$y - k = a(x - h)^2$	$x - h = a(y - k)^2$
Vertical parabola	Horizontal parabola
Vertex at (h,k)	Vertex at (h,k)

A parabola is somewhat like an open ellipse.





EXAMPLE

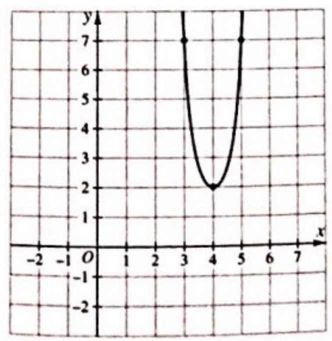
Graph the equation. $y = 2 + 5x^2 - 40x + 80$

Write the equation in standard form.

Subtract 2.	$y - 2 = 2 - 2 + 5x^2 - 40x + 80$
Factor 5 out of the expression on the right side of the equation.	$y - 2 = 5(x^2 - 8x + 16)$
Write the perfect square.	$y - 2 = 5(x - 4)^2$

This is the standard form for a vertical parabola.

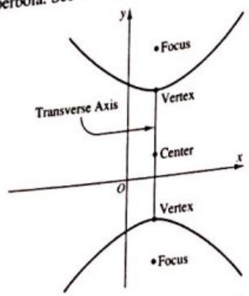
The equation $y - 2 = 5(x - 4)^2$ is a vertical parabola with a vertex at $(4,2)$. Substitute $x = 3$ and $x = 5$ to find two other points on the parabola. $(3,7)$ and $(5,7)$ are other points on the parabola. Graph the equation $y - 2 = 5(x - 4)^2$.



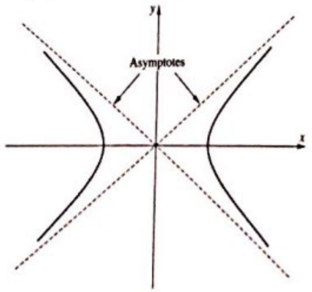
Hyperbola

A hyperbola is the set of all points in the plane for which the difference between the distances to two fixed points, called foci, is constant.

Each hyperbola has two branches. The line segment that connects the two foci intersects the hyperbola at two points, called the vertices. The line segment connecting these vertices is called the transverse axis. The midpoint of the transverse axis is called the center of the hyperbola. See the figure below.



- The general equation of the hyperbola centered at (h,k) is
  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ if the transverse axis is horizontal.
  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ if the transverse axis is vertical.
- If the transverse axis is horizontal, the equation of the axis is $y = k$.
- If the transverse axis is vertical, the equation of the axis is $x = h$.
- The vertices are a units from the center along the transverse axis.
- The distance along the transverse axis from the center to a focus is c, where $c^2 = a^2 + b^2$.
- **Asymptotes** are limit lines approached by the hyperbola.



The equation of the asymptotes is:

Horizontal transverse axis: $y = k - \frac{b}{a}(x-h)$ and $y = k + \frac{b}{a}(x-h)$

Vertical transverse axis: $y = k - \frac{a}{b}(x-h)$ and $y = k + \frac{a}{b}(x-h)$

MODEL ACT PROBLEMS

1. Given the equation of a circle $(x-3)^2 + (y+5)^2 = 16$, what is the sum of the x-coordinate of the center, the y-coordinate of the center, and the radius of the circle?

A. 24
B. 18
C. 12
D. 6
E. 2

SOLUTION

Write the equation in standard form.

$(x-h)^2 + (y-k)^2 = r^2$

$(x-3)^2 + (y+5)^2 = 16$

$\rightarrow (x-3)^2 + (y-(-5))^2 = 4^2$

Therefore, $h = 3$, $k = -5$, and $r = 4$.

Find the sum of h, k, and r.

$3 + (-5) + 4 = 2$

The correct answer is E.

2. What are the coordinates of the foci of an ellipse with the equation $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$?

F. $(-1, 2+\sqrt{5})$ and $(-1, 2-\sqrt{5})$
G. $(-1+\sqrt{5}, 2)$ and $(-1-\sqrt{5}, 2)$
H. $(2, -1+\sqrt{5})$ and $(2, -1-\sqrt{5})$
J. $(1, -2+\sqrt{5})$ and $(1, -2-\sqrt{5})$
K. $(1+\sqrt{5}, -2)$ and $(1-\sqrt{5}, -2)$

SOLUTION

To identify the major axis and the center, write the equation in standard form,

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$$

$$\frac{(x-(-1))^2}{3^2} + \frac{(y-2)^2}{2^2} = 1$$

So, $a = 3$ and $b = 2$. Since $a > b$, the major axis is horizontal. The center of the ellipse is $(-1,2)$. The foci of the ellipse are $\pm\sqrt{|3^2 - 2^2|} = \pm\sqrt{9-4} = \pm\sqrt{5}$ units along the major axis from the center, or at the points $(-1+\sqrt{5}, 2)$ and $(-1-\sqrt{5}, 2)$.

The correct answer is G.

3. What is the vertex of the parabola with equation $y - 5 = 2x^2 - 12x + 18$?

A. $(-3,5)$
B. $(3,-5)$
C. $(5,3)$
D. $(3,5)$
E. $(-3,-5)$

SOLUTION

Write the equation in standard form.

$y - k = a(x-h)^2$

$y - 5 = 2x^2 - 12x + 18$

Factor 2 out of the expression on the right side of the equation.

$y - 5 = 2(x^2 - 6x + 9)$

Write the perfect square.

$y - 5 = 2(x-3)^2$

$h = 3$ and $k = 5$ so the vertex is $(3,5)$.

The correct answer is D.

4. What is the center of the hyperbola with equation $\frac{x^2}{4} - \frac{y^2}{49} = 1$?

F. $(-2,0)$
G. $(2,0)$
H. $(0,0)$
J. $(\sqrt{53},0)$
K. $(-\sqrt{53},0)$

SOLUTION

The general equation for a hyperbola with its center at (h,k) is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

In the given equation, $h = 0$ and $k = 0$, therefore the center is $(0,0)$.

The correct answer is H.

Practice

1. Find the center and the radius of the circle represented by the equation $(x + 2)^2 + (y - 3)^2 = 16$.
2. Complete the square for the equation $x^2 + 4x = 0$.
3. Write the equation for this circle in standard form. $x^2 + y^2 - 10x + 14y - 26 = 0$
4. Graph the circle represented by the equation. $x^2 + y^2 - 4x + 12y + 24 = 0$

The equation of an ellipse is $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$.

5. What is the center of the ellipse?
6. Is the major axis horizontal or vertical?
7. What are the coordinates of the foci of this ellipse?

The equation of an ellipse is $\frac{(x+1)^2}{25} + \frac{(y-2)}{16} = 1$.

8. At what points does this ellipse cross the major and minor axes?
9. What are the coordinates of the foci of this ellipse?
10. Graph the ellipse represented by the equation.

Is each parabola vertical or horizontal?

11. $y - 16 = (x + 6)^2$
12. $x + 25 = 2(y - 7)^2$
13. $x - 34 = 3(y + 14)^2$
14. $y - 8 = 10(x - 12)^2$

The equation of a parabola is $y = 4 + 6x^2 - 36x + 54$.

15. What is the standard equation of the parabola represented by the equation?
16. What is the vertex of the parabola?
17. Determine the vertices of the hyperbola having the equation $\frac{(y+3)^2}{36} - \frac{(x-1)^2}{9} = 1$.
18. Determine the foci of the hyperbola having the equation $\frac{(x+3)^2}{8} - \frac{(y-2)^2}{16} = 1$.
19. Graph the hyperbola having the equation $\frac{x^2}{25} - \frac{y^2}{9} = 1$.
20. Graph the parabola represented by the equation $y + 3 = 2(x - 2)^2$.
21. Graph the parabola represented by the equation $x - 2 = 3(y + 1)^2$.
22. Graph the circle represented by the equation $(x + 5)^2 + (y - 3)^2 = 9$.
23. Graph the ellipse represented by the equation $\frac{(x+1)^2}{16} + \frac{(y+6)^2}{4} = 1$.

24. Graph the hyperbola having the equation $\frac{y^2}{4} - \frac{x^2}{16} = 1$.

(*Answers on page 372*)

25. Graph the hyperbola having the equation $\frac{(x-1)^2}{25} - \frac{(y-2)^2}{36} = 1$.

ACT-TYPE PROBLEMS

1. What is the center of a circle with equation $x^2 - 6x + y^2 + 2y + 9 = 0$?
   - **A.** (3,1)
   - **B.** (1,3)
   - **C.** (−3,1)
   - **D.** (3,−1)
   - **E.** (−1,−3)

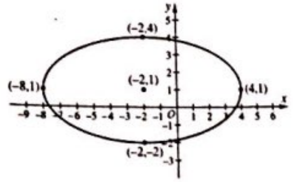
2. What is the equation of the major axis of the ellipse with equation $\frac{(x-4)^2}{49} + \frac{(y+7)^2}{36} = 1$?
   - **F.** $y = -7$
   - **G.** $x = -4$
   - **H.** $y = 4$
   - **J.** $x = 7$
   - **K.** $y = 7$

3. What is the sum of the x-coordinate and the y-coordinate of the vertex of the parabola with equation $y = x^2 - 8x + 10$?
   - **A.** 4
   - **B.** 2
   - **C.** −2
   - **D.** −4
   - **E.** −6

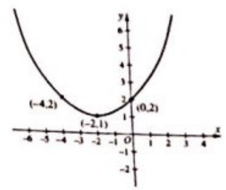
4. Which is the equation of an ellipse?
   - **F.** $8x^2 - 8y^2 = 16$
   - **G.** $(x - 3)^2 + (y + 7)^2 = 3$
   - **H.** $4x^2 + 9y^2 = 36$
   - **J.** $y - 1 = 5(x + 4)^2$
   - **K.** $3x^2 - 4y^2 = 12$

5. Which is the graph of $4y^2 + 8y - 9x^2 + 18x = 41$?

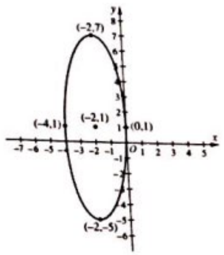
A.



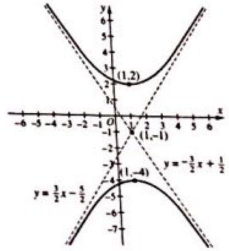
B.



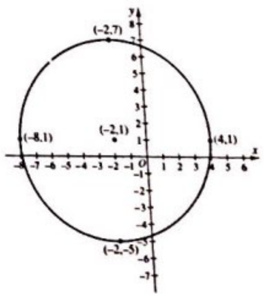
C.



D.



E.



(*Answers on page 373*)

MODEL ACT PROBLEM

Which of the following statements is NOT true?

A. A square is a rectangle.
B. A square is a rhombus.
C. A rectangle is a parallelogram.
D. Adjacent sides of a rhombus are perpendicular.
E. Opposite sides of a parallelogram are congruent.

SOLUTION

Adjacent sides of a rhombus do not have to be perpendicular. Some rhombuses, called squares, do have perpendicular adjacent sides. But the statement is false because it is not true for all rhombuses.

The correct answer is D.

Practice

1. A line segment is drawn connecting any two points in a quadrilateral. What determines whether the figure is concave or convex?
2. In a quadrilateral, two sides are parallel but are not congruent. What type of quadrilateral must this be?
3. For which types of special quadrilaterals can the sum of two opposite angles be less than 180°?
4. What quadrilateral has diagonals that are perpendicular and congruent?
5. In parallelogram *ABCD*, the measure of $\angle B$ is 90°. What type of parallelogram is this?
6. What is the name for a parallelogram that has perpendicular diagonals?
7. What is the name for a parallelogram with congruent diagonals?
8. What properties does a rhombus share with a square?
9. What quadrilateral with a pair of parallel sides is not also a parallelogram?
10. What is the name for a quadrilateral with bisecting diagonals?

(*Answers on page 383*)

ACT-TYPE PROBLEMS

1. An artist wants to draw one geometric shape that has the qualities of two geometric figures. Which of the following shapes could he draw?

   A. A rectangle that is concave
   B. A square that is a trapezoid
   C. A trapezoid that is a parallelogram
   D. A square that is a parallelogram
   E. A parallelogram that is a plane

2. Which of the following is a property of a rectangle?

   F. All four sides are congruent.
   G. Consecutive angles are complementary.
   H. Diagonals are congruent.
   J. Diagonals are perpendicular.
   K. It is a concave quadrilateral.

3. What is the best name for a parallelogram that has four congruent angles and perpendicular diagonals?

   A. Quadrilateral
   B. Parallelogram
   C. Rectangle
   D. Rhombus
   E. Square

4. Consecutive angles in a rhombus must be:

   F. Congruent
   G. Supplementary
   H. Complementary
   J. Right
   K. Acute

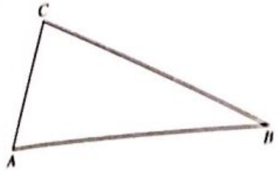
5. The perimeter of a figure is 4 times the length of one side. This figure could be:

   A. A square
   B. A rhombus
   C. A trapezoid
   D. All of the above
   E. None of the above

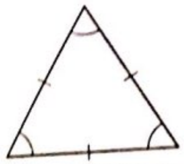
(*Answers on page 383*)

General Properties of Triangles

A triangle is a three-sided polygon.



Triangle *ABC*



Equilateral triangle

An **equilateral triangle** has three congruent angles and three congruent sides.

Isosceles triangle

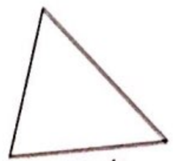
An **isosceles triangle** has two congruent angles, called base angles, and two congruent sides. The third angle is called the vertex angle. The third side is called the base.

Scalene triangle

A **scalene triangle** has all different size angles and all different length sides.

Right triangle

A **right triangle** has one right angle.



Acute triangle

An **acute triangle** has all acute angles.

Obtuse triangle

An **obtuse triangle** has one obtuse angle.

Triangle Facts

- The sum of the angles in a triangle is 180°.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

MODEL ACT PROBLEM

Which of the following statements about triangles is true?

A. The sum of the measures of the angles of a triangle is greater than or equal to 180°.

B. It is possible to have the sum of the lengths of two sides of a triangle be equal to the length of the third side.

C. Triangles always have an angle whose measure is greater than or equal to 90°.

D. A triangle can have more than one right angle.

E. If a triangle has two sides of equal length, then it must have two angles of equal measure.

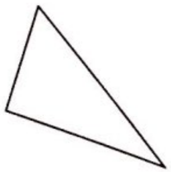
SOLUTION

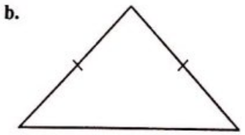
Only the final statement is true. All of the others are false.

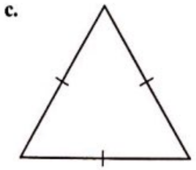
The correct answer is E.

Practice

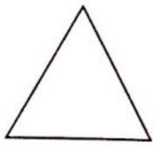
1. Indicate whether each triangle is equilateral, isosceles, or scalene.

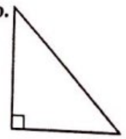
a.


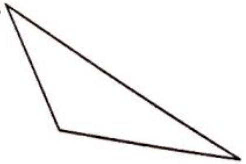
b.


c.


2. Indicate whether each triangle is right, acute, or obtuse.

a.


b.


c.


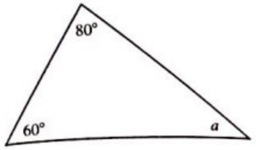
3. What is the measure of each angle in an equilateral triangle?

4. One base angle in an isosceles triangle measures 50°. What are the measures of the other angles?

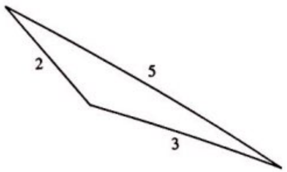
5. The vertex angle in an isosceles triangle measures 50°. What are the measures of the base angles?

6. If one angle of a scalene triangle measures 10°, then what measure is impossible for either of the other two angles?

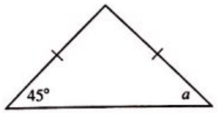
7. What is the measure of $\angle a$?



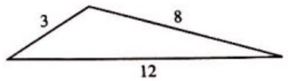
8. Are the measurements of this triangle realistic? If not, explain why.

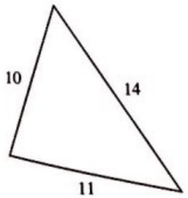


9. What is the measure of $\angle a$?



Are the measurements of these triangles realistic? Explain.

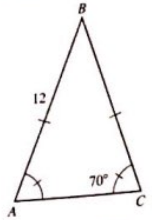
10.


11.


(Answers on page 383)

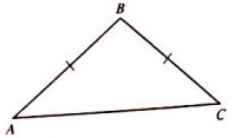
ACT-TYPE PROBLEMS

1. Which of the following statements can NOT be concluded from the information presented in the figure shown below?



A. m∠$B = 40°$
B. m∠C + m∠$B = 100°$
C. $AB + BC = 24$
D. m∠$A = 70°$
E. $BC = 12$

2. In the isosceles triangle below, $AB = CB$. What is the measure of the vertex angle if m∠$A = 40°$?



F. 40°
G. 50°
H. 90°
J. 100°
K. 140°

3. Which of the following choices could NOT be the lengths of the sides of a triangle?

A. 3, 5, 9
B. 6, 7, 12
C. 2, 6, 6
D. 12, 12, 12
E. 5, 12, 13

4. Which of the following statements is FALSE?

F. 40°, 60°, and 80° are the angle measures of some scalene triangles.
G. 60°, 60°, and 60° are the angle measures of some acute triangles.
H. 60°, 60°, and 60° are the angle measures of equilateral triangles.
J. 45°, 45°, and 90° are the angle measures of some obtuse triangles.
K. 45°, 45°, and 90° are the angle measures of some isosceles triangles.

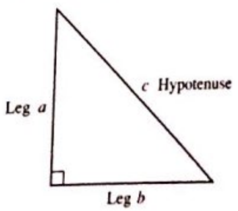
5. Which of the following types of triangle always has at least two angles with equal measures?

A. Scalene triangle
B. Obtuse triangle
C. Equilateral triangle
D. Right triangle
E. Acute triangle

(Answers on page 384)

Right Triangles

The two sides of a right triangle that meet at the right angle are called legs. The third side is called the hypotenuse.



The Pythagorean Theorem

The Pythagorean Theorem describes the relationship among the three sides of a right triangle.

$a^2 + b^2 = c^2$

You should memorize this theorem.

EXAMPLES

1. The lengths of the legs of a right triangle are 3 and 4. What is the length of the hypotenuse?

Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$

$3^2 + 4^2 = c^2$

$9 + 16 = c^2$

$25 = c^2$

$5 = c$

The length of the hypotenuse is 5.

2. The length of one leg of a right triangle is 4. The length of the hypotenuse is $\sqrt{65}$. How long is the other leg?

Use the Pythagorean Theorem.

$a^2 + b^2 = c^2$

$4^2 + b^2 = (\sqrt{65})^2$

$16 + b^2 = 65$

$b^2 = 65 - 16 = 49$

$b = 7$

The length of the other leg is 7.

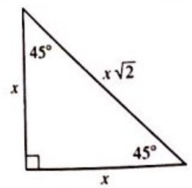
Whole numbers that satisfy the Pythagorean Theorem are called **Pythagorean triples.** The most famous Pythagorean triple is 3, 4, 5. Memorize it. The multiples of Pythagorean triples are also Pythagorean triples. Here are the first four triples along with the first three multiples of the 3-4-5 triple.

3, 4, 5	5, 12, 13	7, 24, 25	8, 15, 17
6, 8, 10			
9, 12, 15			
12, 16, 20			

Remember: The first two numbers are the lengths of the legs. The third number is the length of the hypotenuse.

Isosceles Right Triangle

Both legs of an isosceles right triangle are the same length. The length of the hypotenuse is $\sqrt{2}$ times the length of a leg. The angle measures for this triangle are 45°, 45°, and 90°.



In an isosceles right triangle, you can quickly find the lengths of all the sides once you know the length of any one side.

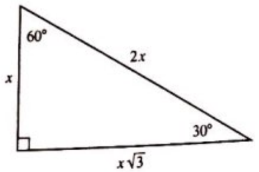
EXAMPLES

1. The length of the leg of an isosceles right triangle is 5. How long is the hypotenuse?
   The hypotenuse is $5\sqrt{2}$.
2. In an isosceles right triangle, the length of the hypotenuse is 14. How long are the legs?
   Divide 14 by $\sqrt{2}$: $\frac{14}{\sqrt{2}} = \frac{14}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$
   The length of each leg is $7\sqrt{2}$.

There should not be a radical in the denominator.

30–60–90 Triangle

In this type of triangle, the hypotenuse is twice the length of the shorter leg. The longer leg is $\sqrt{3}$ times the shorter leg.



EXAMPLE

The longer leg of a 30–60–90 triangle is 21. What is the length of the hypotenuse?

Divide 21 by $\sqrt{3}$ to find the length of the shorter leg. $\frac{21}{\sqrt{3}} = \frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} = 7\sqrt{3}$

Multiply the shorter leg by 2 to find the length of the hypotenuse.

$2 \times 7\sqrt{3} = 14\sqrt{3}$

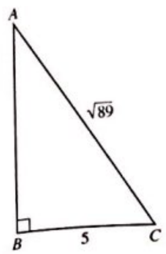
The length of the hypotenuse is $14\sqrt{3}$.

CALCULATOR TIP

If an ACT question about special triangles has radicals in the answer choices, check to see if your calculator can display answers in radical form. Otherwise, don't use a calculator for computation.

MODEL ACT PROBLEM

What is the length of leg *AB* in the triangle shown below?



A. 5
B. 7
C. 8
D. 9.5
E. 10.7

SOLUTION

Use the Pythagorean Theorem to find the measure of *AB*.

$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$

$a^2 + b^2 = c^2$

$5^2 + b^2 = (\sqrt{89})^2$

$25 + b^2 = 89$

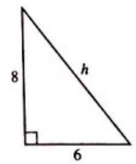
$b^2 = 64$

$b = 8$

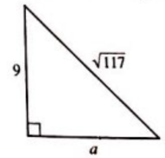
The correct answer is C.

Practice

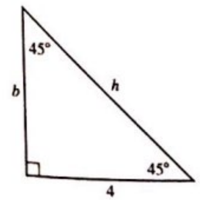
1. What are the measures of the angles in an isosceles right triangle?
2. Use the Pythagorean Theorem to find the measure of the hypotenuse.



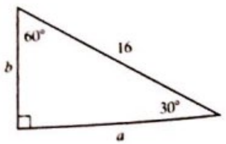
3. What is the length of the missing leg?



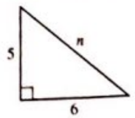
4. This is an isosceles right triangle. What are the lengths of the missing leg and the hypotenuse?



5. This is a 30–60–90 triangle. What are the lengths of legs a and b?

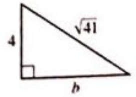


6. Use the Pythagorean Theorem to find the length of the hypotenuse.

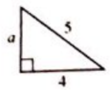


In 7 and 8, use the Pythagorean Theorem to find the lengths of the missing legs.

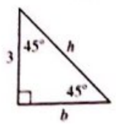
7.



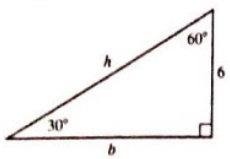
8.



9. The figure below is an isosceles right triangle. What are the lengths of the missing leg and the hypotenuse?



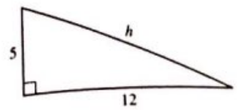
10. The figure below is a 30–60–90 triangle. What are the lengths of the missing leg and the hypotenuse?



(Answers on page 384)

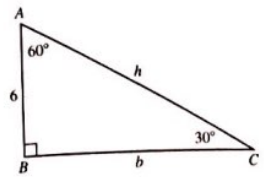
ACT-TYPE PROBLEMS

1. What is the length of the hypotenuse of the triangle shown below?



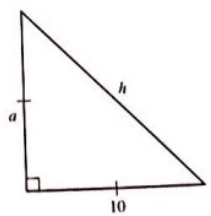
A. 5
B. 12
C. 13
D. 17
E. 25

2. What is the product of b and h in the triangle shown below?



F. $72\sqrt{3}$
G. 24
H. 216
J. 1,728
K. 432

3. What is the sum of a and h in the figure shown below?



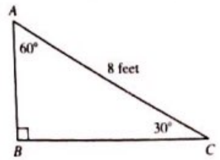
A. $10 + 10\sqrt{3}$
B. $20\sqrt{2}$
C. $10\sqrt{2}$
D. $10 + 10\sqrt{2}$
E. $20\sqrt{3}$

(Answers on page 384)

4. The base of a ladder is placed 10 feet from a house, and the top of the ladder touches the house 14 feet above the ground. If the house creates a 90° angle with the ground, how long is the ladder, rounded to the nearest tenth?

F. 9.8 feet
G. 17.2 feet
H. 19.6 feet
J. 21.0 feet
K. 24.0 feet

5. The triangle below is a 30–60–90 triangle. What is the sum of the lengths of side $\overline{AB}$ and side $\overline{BC}$?

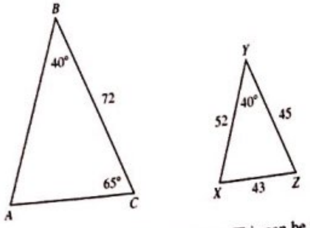


A. 4 feet
B. $4 + 4\sqrt{3}$ feet
C. $4\sqrt{3}$ feet
D. $12 + \sqrt{3}$ feet
E. 12 feet

Similar Triangles

Similar figures have the same shape but not necessarily the same size.

- The lengths of corresponding sides of similar triangles are proportional.
- Corresponding angles of similar triangles are congruent.
- The vertices of similar triangles are listed in the same order. In other words, if triangle ABC is similar to triangle DEF, then $\angle A$ corresponds to $\angle D$, $\angle B$ corresponds to $\angle E$, and $\angle C$ corresponds to $\angle F$. This is true for other congruent figures as well.



Triangles ABC and XYZ are similar. This can be written $\triangle ABC \sim \triangle XYZ$.

Angles

$\angle B$ and $\angle Y$ are corresponding angles. Corresponding angles have equal measures. So $m\angle B = m\angle Y = 40°$. Now we can find the measure of $\angle A$.

The sum of the measures of $\angle B$ and $\angle C$ is 105°.

The sum of all the angle measures in a triangle is 180°.

The measure of $\angle A = 180° - 105° = 75°$.

By visual inspection of these triangles, we see that the following angles are corresponding angles.

$\angle B$ and $\angle Y$	$m\angle B = m\angle Y = 40°$
$\angle A$ and $\angle X$	$m\angle A = m\angle X = 75°$
$\angle C$ and $\angle Z$	$m\angle C = m\angle Z = 65°$

Use the similarity of these triangles to find the missing measurements.

CALCULATOR TIP

Use your calculator to determine or check the proportional relationship between corresponding sides of similar triangles.

Use this proportion to find the lengths of the other sides.

Sides

Corresponding sides are opposite corresponding angles. So $\overline{BC}$ and $\overline{YZ}$ are corresponding sides.

The lengths of corresponding sides are proportional.

$$\frac{BC}{YZ} = \frac{72}{45} = 1.6$$

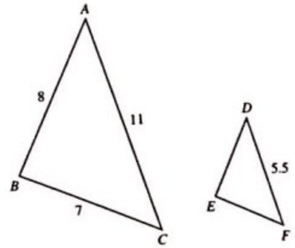
The length of a side of $\triangle ABC$ will be 1.6 times the length of the corresponding side in $\triangle XYZ$.

Therefore we can find the missing lengths of the other corresponding sides.

XZ and AC.	The length of $XZ = 43$. $1.6 \times 43 = 68.8$. The length of AC is 68.8.
XY and AB.	The length of $XY = 52$. $1.6 \times 52 = 83.2$. The length of AB is 83.2.

MODEL ACT PROBLEMS

1. In the figure below, $\triangle ABC$ and $\triangle DEF$ are similar triangles. How many units long is $\overline{DE}$?



A. 2
B. 3
C. 4
D. 5
E. 6

SOLUTION

Since $\triangle ABC$ is similar to $\triangle DEF$, corresponding sides are proportional.

Therefore, $\frac{DE}{AB} = \frac{DF}{AC}$.

Substitute known values. $\frac{DE}{8} = \frac{5.5}{11}$

Multiply by 8. $DE = \frac{5.5}{11} \times 8$

$DE = \frac{1}{2} \times 8 = 4$

The correct answer is C.

2. $\triangle XRK$ is similar to $\triangle STV$, and both triangles are scalene. Which of the following statements is NOT true?

F. $\angle X \cong \angle S$
G. $\angle R \cong \angle V$
H. $\overline{XR}$ and $\overline{ST}$ are corresponding sides.
J. $\frac{KX}{RK} = \frac{VS}{TV}$
K. $\angle X + \angle K = \angle V + \angle S$

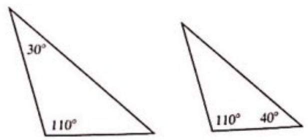
SOLUTION

$\angle R$ and $\angle V$ are not corresponding angles, therefore they are not congruent.

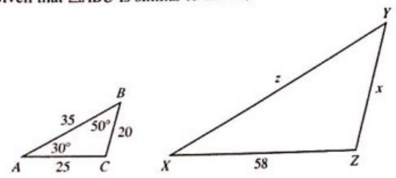
The correct answer is G.

Practice

1. What is true about corresponding angles of similar triangles?
2. What is true about corresponding sides of similar triangles?
3. Are these two triangles similar? Explain why or why not.

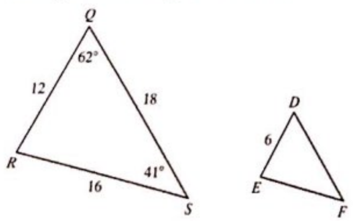


Given that $\triangle ABC$ is similar to $\triangle XYZ$, find the missing measurements of each triangle.



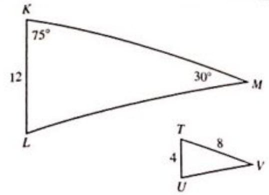
4. What is the length of z?
5. What is the length of x?
6. What is $m\angle C$?
7. What is $m\angle X$?
8. What is $m\angle Z$?
9. What is $m\angle Y$?

In the diagram below, $\triangle QRS$ is similar to $\triangle DEF$.

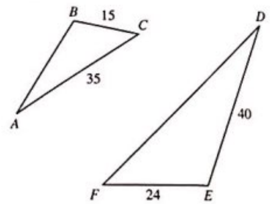


10. What is the measure of $\angle R$?
11. What is the measure of $\angle E$?
12. What is the measure of $\angle D$?
13. What is the measure of $\angle F$?
14. What is the length of EF?
15. What is the length of DF?

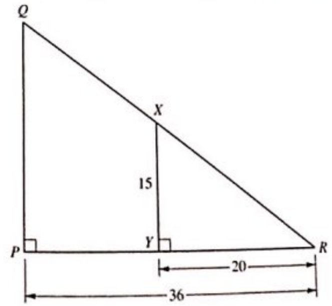
In the diagram below, $\triangle KLM$ is similar to $\triangle TUV$.



16. What is the measure of $\angle L$?
17. What is the measure of $\angle T$?
18. What is the measure of $\angle U$?
19. What is the measure of $\angle V$?
20. What is the length of KM?
21. What is the length of LM?
22. If $\triangle ABC$ below is similar to $\triangle DEF$, how much longer is the perimeter of $\triangle DEF$ than the perimeter of $\triangle ABC$?



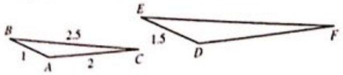
23. If $\triangle PQR$ below is similar to $\triangle YXR$, what are the perimeter and area of each triangle?



(Answers on page 384)

ACT-TYPE PROBLEMS

1. In the figure below, $\triangle ABC$ is similar to $\triangle DEF$. What is the length of $\overline{DF}$?

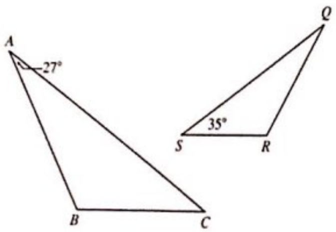


A. 1
B. 2
C. 3
D. 4
E. 5

2. If $\triangle QRS$ is similar to $\triangle XYZ$, which choice gives corresponding sides?

F. $\overline{QR}$ and $\overline{RQ}$
G. $\overline{RS}$ and $\overline{YZ}$
H. $\overline{QS}$ and $\overline{XY}$
J. $\overline{SR}$ and $\overline{SQ}$
K. $\overline{RQ}$ and $\overline{XZ}$

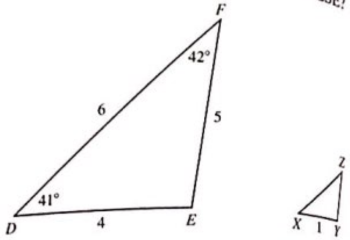
3. In the figure below, $\triangle ABC$ is similar to $\triangle QRS$. What is the sum of the measures of $\angle B$ and $\angle Q$?



A. 54°
B. 62°
C. 145°
D. 153°
E. 180°

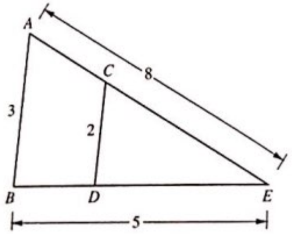
(Answers on page 385)

4. In the figure below, $\triangle DEF$ is similar to $\triangle XYZ$. Which of the following statements is FALSE?



F. $m\angle Z = 42°$
G. $XZ = 1.5$
H. $m\angle X = 41°$
J. $YZ = 1.2$
K. $m\angle Y = 97°$

5. In the figure below, $\triangle ABE$ is similar to $\triangle CDE$. What is the sum of AC and BD?



A. $3\frac{1}{3}$
B. $4\frac{1}{3}$
C. $5\frac{1}{3}$
D. $7\frac{2}{3}$
E. $8\frac{2}{3}$

Concept of Proof and Proof Techniques

Proof means using what is known in a logically convincing way to establish that a hypothesis is true or false. Much of geometry involves proving or disproving hypotheses. Consider this simple example.

EXAMPLE

Given: One angle of a triangle measures 90°, while another angle measures 50°. The sum of the measures of the angles in a triangle is 180°.

Hypothesis: The third angle in the triangle measures 40°.

Proof: $90° + 50° = 140°$ $180° - 140° = 40°$

The proof uses only what is known and is logical.

Congruent Triangles

Many proofs involve showing that two triangles are congruent. We use Side Side Side (SSS), Angle Side Angle (ASA), and Side Angle Side (SAS) congruence relations between triangles to prove that triangles are congruent.

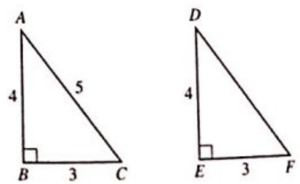
- SSS: If each side of one triangle is congruent to the corresponding side of another triangle, then the two triangles are congruent.
- ASA: In a triangle, if two angles and the side between them are congruent to the corresponding angles and side of another triangle, then the two triangles are congruent.
- SAS: In a triangle, if two sides and the angle between them are congruent to the corresponding sides and angle of a second triangle, then the two triangles are congruent.

Here is a traditional example.

EXAMPLE

Given: 1. The SSS congruence relation between two triangles: If each side of one triangle is congruent to the corresponding side of another triangle, then the two triangles are congruent.

2. The dimensions of the two right triangles below:



3. The Pythagorean Theorem: In a right triangle, the sum of the squares of the legs (a and b) equals the square of the hypotenuse (c). ($a^2 + b^2 = c^2$)

Hypothesis: Triangle ABC is congruent to triangle DEF ($\triangle ABC \cong \triangle DEF$).

Proof: From the diagram: $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$

Use the Pythagorean Theorem to find the length of side DF.

$$a^2 + b^2 = c^2$$
$$(EF)^2 + (DE)^2 = (DF)^2$$
$$3^2 + 4^2 = (DF)^2$$
$$9 + 16 = (DF)^2$$
$$25 = (DF)^2$$
$$5 = DF$$

$\overline{DF}$ and $\overline{AC}$ are both 5 units long, so $\overline{DF} \cong \overline{AC}$.

$\triangle ABC \cong \triangle DEF$ by SSS.

The proof uses only what is known and logically shows that the two triangles are congruent by SSS. (*Note*: The proof could also have been done using SAS.)

Here is a more formal way to present this proof. It presents each step in the proof and gives the reason for that step.

EXAMPLE

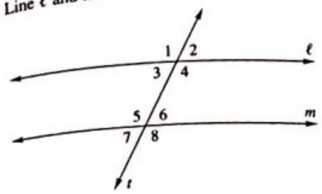
Hypothesis: Triangle ABC is congruent to triangle DEF ($\triangle ABC \cong \triangle DEF$).

Statement	*Reason*
1. $AB = DE = 4$ $BC = EF = 3$ $AC = 5$	1. Given
2. $(EF)^2 + (DE)^2 = (DF)^2$ $3^2 + 4^2 = (DF)^2$ $9 + 16 = (DF)^2$ $25 = (DF)^2$ $5 = DF$	2. Pythagorean Theorem
3. $AC = DF$	3. These two sides are both 5 units long.
4. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$	4. Segments of equal length are congruent.
5. $\triangle ABC \cong \triangle DEF$	5. SSS congruence theorem

MODEL ACT PROBLEM

Look at the figure below.

Line ℓ and line m are parallel, and line t is a transversal that cuts through ℓ and m.



Which of the following statements is FALSE?

A. $\angle 6 \cong \angle 7$

B. $\angle 6 \cong \angle 3$

C. $\angle 8 \cong \angle 4$

D. $\angle 1 \cong \angle 7$

E. $\angle 2 \cong \angle 7$

SOLUTION

We know quite a few things about parallel lines cut by a transversal.

Corresponding angles are congruent. The corresponding angles in this figure are: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$.

Alternate interior angles are congruent. The pairs of alternate interior angles in this figure are: $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$.

Alternate exterior angles are congruent. The pairs of alternate exterior angles in this figure are: $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$.

Vertical angles are congruent. The pairs of vertical angles in this figure are: $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, $\angle 5$ and $\angle 8$, $\angle 6$ and $\angle 7$.

Statement		**Reason**
A. $\angle 6 \cong \angle 7$	True	vertical angles
B. $\angle 6 \cong \angle 3$	True	alternate interior angles
C. $\angle 8 \cong \angle 4$	True	corresponding angles
D. $\angle 1 \cong \angle 7$	?	does not match known information
E. $\angle 2 \cong \angle 7$	True	alternate exterior angles

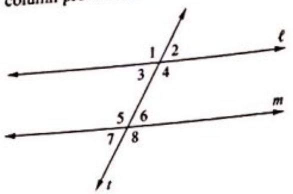
All the other statements were proved true, so statement D must be false.

$\angle 1$ is not congruent to $\angle 7$.

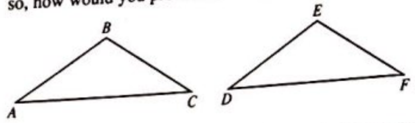
The correct answer is D.

Practice

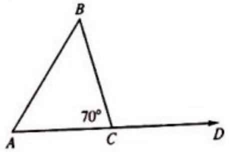
1. Line t is a transversal intersecting lines ℓ and m. Given that $\angle 2 \cong \angle 6$, write a two-column proof that shows that $\angle 3 \cong \angle 6$.



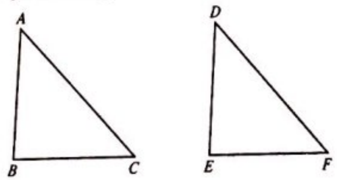
2. In the diagram below, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$. Is $\triangle ABC \cong \triangle DEF$? If so, how would you prove them congruent?



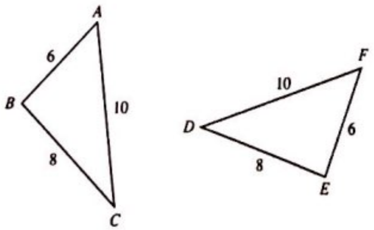
3. Given that $m\angle BCA = 70°$ in the diagram below, write a two-column proof that proves $m\angle BCD = m\angle A + m\angle B$.



4. In $\triangle ABC$ and $\triangle DEF$ below, $\angle C \cong \angle F$. What other piece(s) of information would you need to prove that $\triangle ABC \cong \triangle DEF$ by ASA congruence?

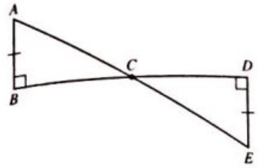


5. In the figure below, name the congruent triangles.

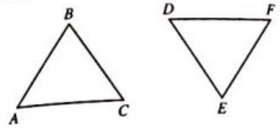


6. Given two triangles, $\triangle ABC$ and $\triangle DEF$, where $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. Is $\triangle ABC \cong \triangle DEF$? Explain why or why not.

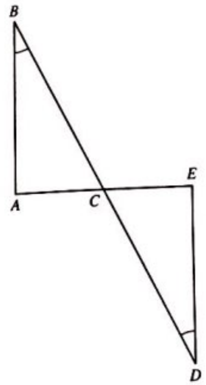
7. In the figure shown below, is $\triangle ABC \cong \triangle EDC$? Explain why.



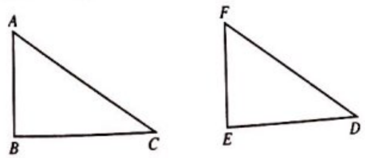
8. Given: $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, and $\overline{BC} \cong \overline{EF}$ in the figure shown.
How would you explain that $\angle A \cong \angle D$?



9. Which corresponding angles or corresponding sides in the triangles below could be congruent but not help to prove that the triangles are congruent?



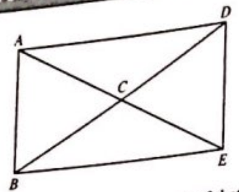
10. Given: $\angle A \cong \angle F$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle D$. Are triangles ABC and FED in the figure below congruent? Prove they are congruent or explain why they are not congruent.



(Answers on page 385)

ACT-TYPE PROBLEMS

1.



Below is a two-column proof that m∠CBA + m∠ACB = m∠CDE + m∠DCE in the figure above.

Statement	*Reason*
1. ?	1. Given
2. m∠ACB = m∠DCE	2. Vertical angles have equal measures.
3. m∠CBA + m∠ACB = m∠CDE + m∠DCE	3. When equals are added to equals, the sums are equal.

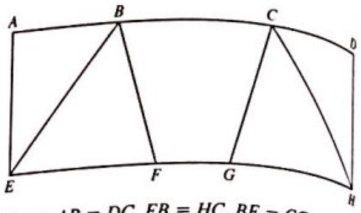
Which of the following statements must be included in Statement 1 so that the proof is complete?

A. m∠ABE = m∠DEB
B. m∠CBA = m∠CDE
C. m∠ECB = m∠ACD
D. m∠EAB = m∠BDE
E. m∠ECA = m∠BCD

2. If you are given that △ABC ≅ △EDF, which of the following statements is NOT known?

F. $\overline{AC} \cong \overline{EF}$
G. ∠ABC ≅ ∠FDE
H. $\overline{CA} \cong \overline{FD}$
J. ∠C ≅ ∠F
K. △BCA ≅ △DFE

3.



Given: AB = DC, EB = HC, BF = CG, and ∠EAB and ∠HDC are right angles.

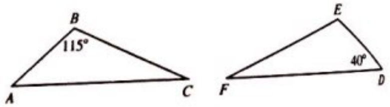
Which of the following can NOT be proved about the figure above?

A. AC = BD
B. △ABE ≅ △DCH
C. AE = DH
D. EF = GH
E. m∠ABE = m∠DCH

4. Given two triangles where △ABC ≅ △DEF, which of the following could NOT be proved congruent?

F. ∠B ≅ ∠E
G. $\overline{AC} \cong \overline{FD}$
H. $\overline{AB} \cong \overline{EF}$
J. ∠C ≅ ∠F
K. $\overline{DE} \cong \overline{ED}$

5. Given △ABC ≅ △DEF, what is the measure of ∠C in the figure below?

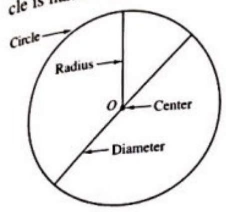


A. 115°
B. 75°
C. 65°
D. 40°
E. 25°

(*Answers on page 386*)

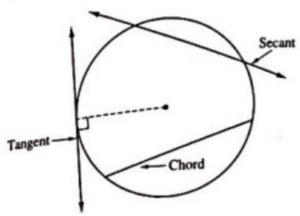
Circles

A **circle** is all the points a set distance from a fixed point called the center. The diagram below shows a circle along with its center, a radius, and a diameter. A **diameter** is a line segment that passes through the center and has its endpoints on the circle. A **radius** is a line segment with the center for one endpoint and the other endpoint on the circle. A circle is named by its center. The circle below is circle *O*.



The circle is just the points equidistant from the center. The circle does not include any interior points.

Chords, Secants, and Tangents



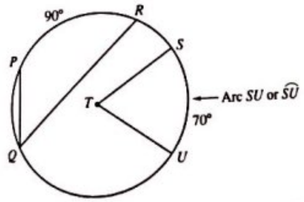
A **chord** is a line segment with endpoints on the circle. The diameter is the longest chord.

A **secant** is a line that contains a chord.

A **tangent** is a line that touches exactly one point on the circle.

The tangent is perpendicular to the radius at the point of tangency.

Arcs and Angles



An **arc** is any portion of the circle. $\overset{\frown}{SU}$ is an arc.

A **central angle** is an angle formed by two radii. ∠STU is a central angle. ∠STU intercepts $\overset{\frown}{SU}$.

An **inscribed angle** is an angle formed by two chords with the vertex on the circle. ∠PQR is an inscribed angle. ∠PQR intercepts $\overset{\frown}{PR}$.

Angle and Arc Measures

The total degree measure of a circle is 360°.

- The measure of a central angle equals the measure of the intercepted arc.

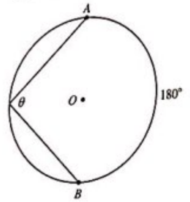
  In the previous figure, $m\widehat{SU} = 70°$, $m\angle STU = 70°$.

- The measure of an inscribed angle equals $\frac{1}{2}$ the measure of the intercepted arc.

  In the previous figure, $m\widehat{PR} = 90°$, $m\angle PQR = 45°$.

MODEL ACT PROBLEMS

1. In the figure below, the measure of $\widehat{AB}$ is 180°. What is the measure of the inscribed angle θ?



A. 30°
B. 45°
C. 60°
D. 90°
E. 180°

SOLUTION

An inscribed angle is half the measure of the intercepted arc. Therefore:

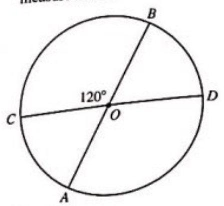
$m\angle\theta = \frac{1}{2}(m\widehat{AB})$

$m\angle\theta = \frac{1}{2}(180°)$

$m\angle\theta = 90°$

The correct answer is D.

2. In the circle below with center O, what is the measure of $\widehat{BD}$?



F. 30°
G. 45°
H. 60°
J. 120°
K. 180°

SOLUTION

Segment $\overline{CD}$ is a diameter.

$m\angle BOC + m\angle BOD = 180°$

$m\angle BOD = 180° - 120°$

$m\angle BOD = 60°$

The measure of a central angle is equal to the measure of the arc that it intercepts.

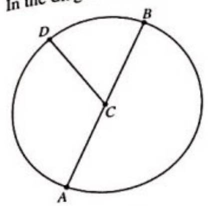
Therefore $m\angle BOD = m\widehat{BD}$.

$60° = m\widehat{BD}$

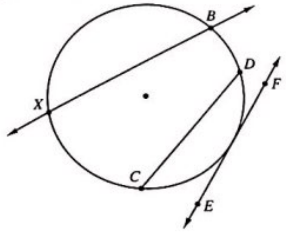
The correct answer is H.

Practice

1. What is a diameter?
2. What is a radius?
3. In the diagram below, identify the center, radius, and diameter.

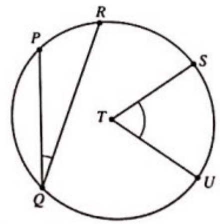


4. What is a chord?
5. What is a secant?
6. What is a tangent to a circle?
7. In the diagram below, identify a chord, secant, and tangent.



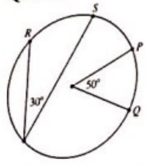
8. What is a central angle of a circle?
9. What is an inscribed angle?

Questions 10 and 11 refer to the figure below.



10. Identify the central angle and the inscribed angle.
11. What arcs are intercepted by the inscribed angle and the central angle?

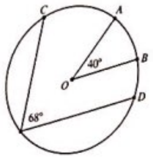
Questions 12 and 13 refer to the figure below.



12. What is the measure of $\overparen{PQ}$?

13. What is the measure of $\overparen{RS}$?

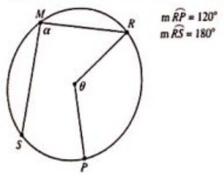
Questions 14 and 15 refer to the figure below.



14. What is the measure of $\overparen{AB}$?

15. What is the measure of $\overparen{CD}$?

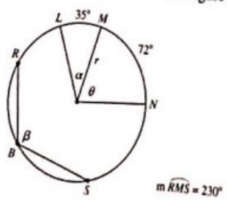
Questions 16 and 17 refer to the figure below.



16. What is the measure of ∠θ?

17. What is the measure of ∠α?

Questions 18–20 refer to the figure below.



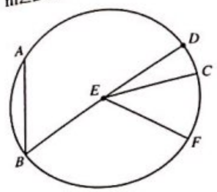
18. What is the measure of ∠θ?

19. What is the measure of ∠α?

20. What is the measure of ∠β?

21. Circle E has a circumference of 6 units.

$m\angle DEF = 80°$ $m\angle CEF = 55°$ $m\angle ABD = 60°$



What is the shortest distance around the circle from:

a. Point A to point D

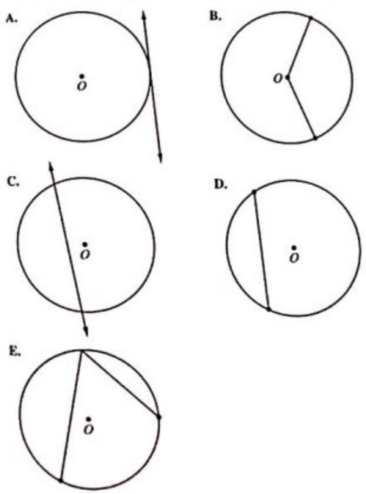
b. Point D to point F

c. Point B to point C

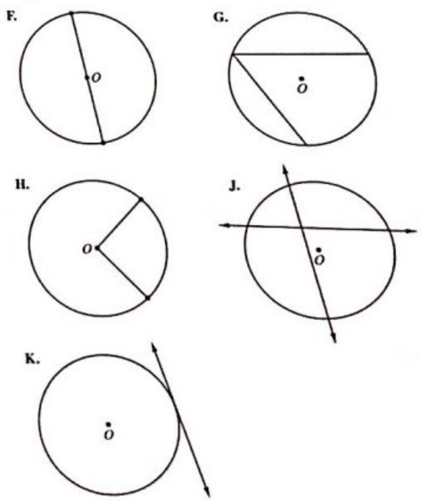
(Answers on page 386)

ACT-TYPE PROBLEMS

1. Which of the figures below shows a chord?

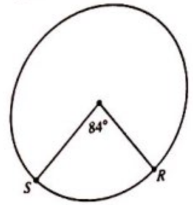


2. Which of the figures below shows an inscribed angle?



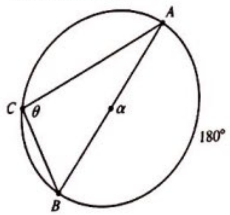
art
Griffin Ray

3. In the figure below, what is the measure of arc RS?



A. 16°
B. 36°
C. 42°
D. 84°
E. 168°

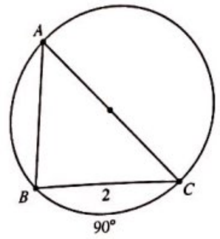
4. In the figure below, $\overset{\frown}{AB}$ measures 180° and both $\angle\theta$ and $\angle\alpha$ intercept this arc. What is the sum of $m\angle\theta$ and $m\angle\alpha$?



F. 45°
G. 90°
H. 145°
J. 180°
K. 270°

(*Answers on page 387*)

5. In the figure below, the measure of $\overset{\frown}{AC}$ is equal to twice the measure of $\overset{\frown}{BC}$. What is the length of segment AC in $\triangle ABC$?



A. 2
B. $2\sqrt{2}$
C. $2\sqrt{3}$
D. 4
E. 6

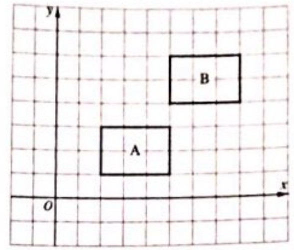
▪ Transformations in the Plane

A **transformation** in the plane means shifting a figure by sliding, flipping, or rotating it from one location to another.

Translation

A **translation** in the plane means sliding a figure from its original position to another position.

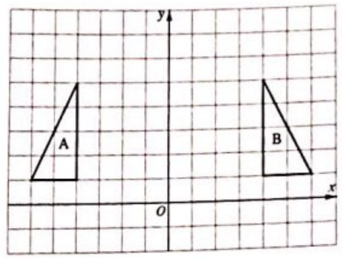
Rectangle A is the original figure. Sliding rectangle A 3 units up and 3 units to the right formed rectangle B.



Reflection

A **reflection** in the plane means flipping a figure across a line such as the x-axis or the y-axis.

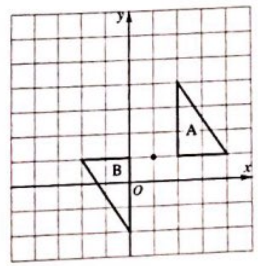
Triangle A is the original triangle. Triangle B is formed by flipping triangle A across the y-axis.



Rotation

A **rotation** in the plane means rotating a figure about a point in the plane.

Triangle A is the original triangle. Triangle B is formed by rotating triangle A 180° about the point (1,1).



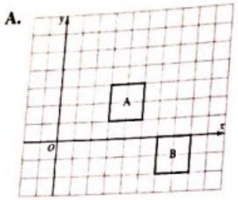
CALCULATOR TIP

Some graphing calculators can graph geometric figures and do transformations.

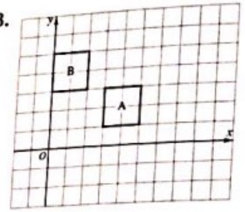
MODEL ACT PROBLEM

Given square A in the graph, which of the following choices shows that square B is obtained by sliding 3 units left and 3 units up?

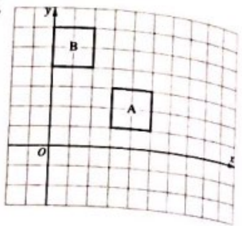
A.



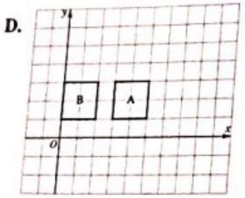
B.



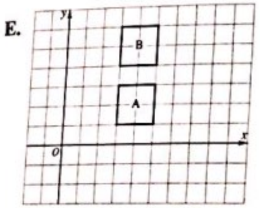
C.



D.



E.



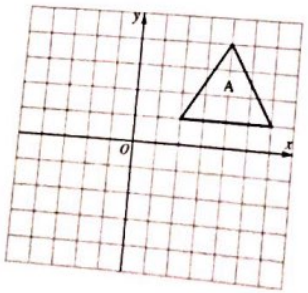
SOLUTION

In choice C, square B is formed from square A by a horizontal translation of 3 to the left and a vertical translation of 3 up.

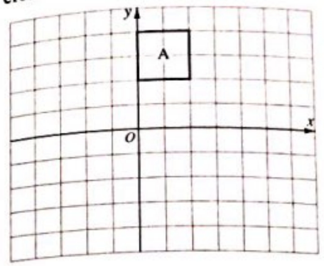
The correct answer is C.

Practice

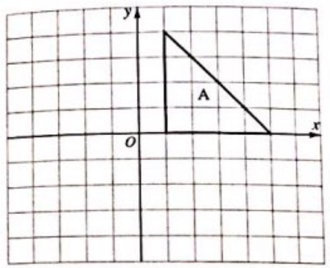
1. Using triangle A as your starting triangle, create a triangle B by flipping triangle A across the x-axis.



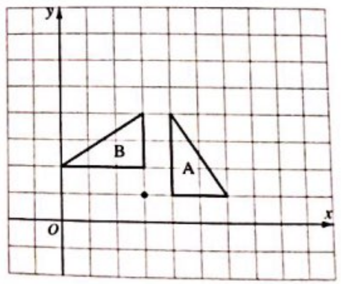
2. Using square A as your starting square, rotate the figure 180° about the origin to create square B.



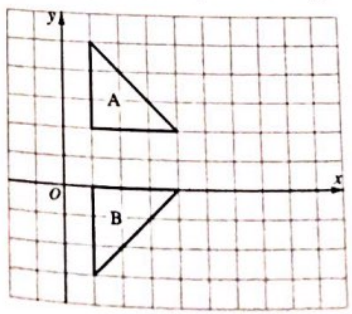
3. Using triangle A as your starting triangle, create triangle B by sliding triangle A 2 units down and 3 units to the left.



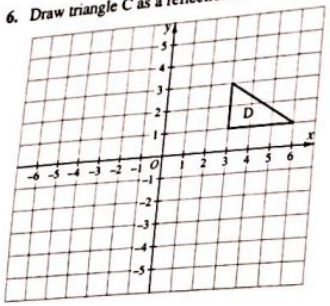
4. Look at the diagram below. How is triangle B obtained from triangle A?



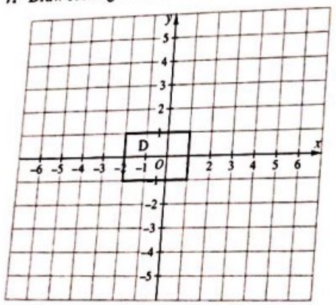
5. In the diagram below, how is triangle B obtained from triangle A?



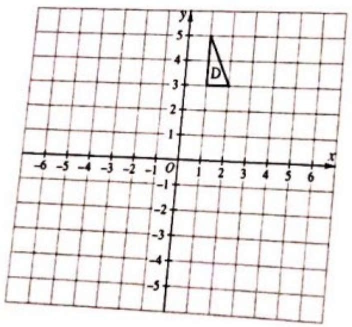
6. Draw triangle C as a reflection of triangle D across the line $x = 2$.



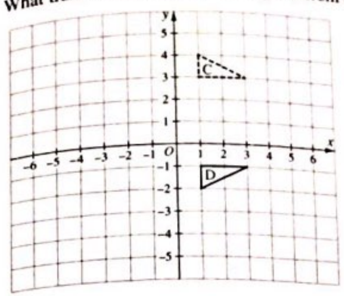
7. Draw rectangle C as a rotation of rectangle D by 180° about the point $(-2,1)$.



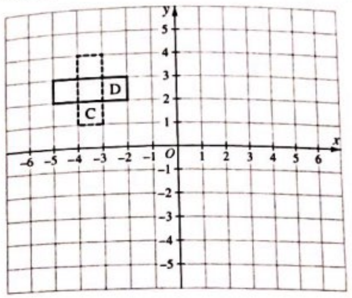
8. Draw triangle C by translating triangle D down 2 units and then reflecting it across the y-axis.



9. What transformation creates triangle C from triangle D in the diagram below?



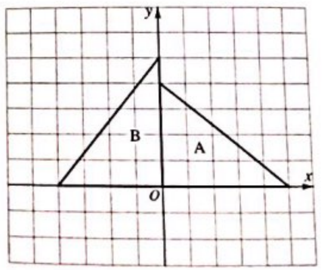
10. What transformation creates rectangle C from rectangle D in the diagram below?



(Answers on page 387)

ACT-TYPE PROBLEMS

1. Which of the following translations produced triangle B from triangle A in the diagram below?

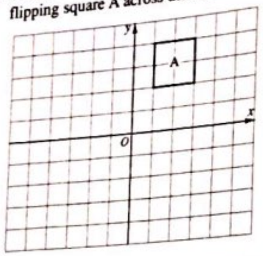


A. Horizontal shift 4 units to the left
B. Flip across the y-axis
C. 90° rotation about the origin
D. Flip across the x-axis
E. 60° rotation about the origin

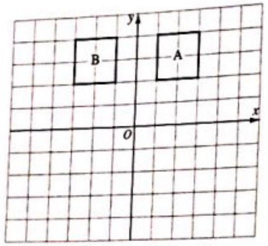
2. If the center of a circle is the origin, which of the following translations will NOT place the circle on top of itself?

F. Horizontal translation to the left 1 unit, followed by a horizontal translation to the right 1 unit
G. Any rotation about the origin
H. A reflection across the x-axis
J. A vertical translation up 2 units, followed by a vertical translation up 2 units
K. A reflection across the y-axis

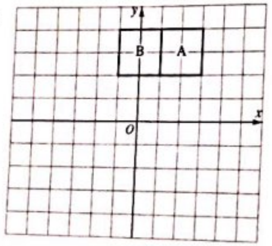
3. Which of the following creates a square B by flipping square A across the line $x = 1$?



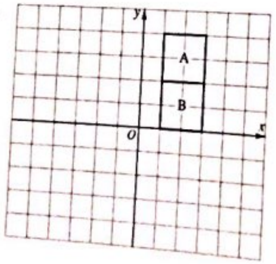
A.



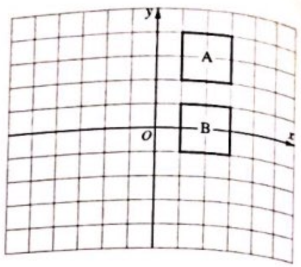
B.



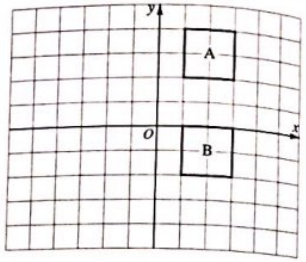
C.



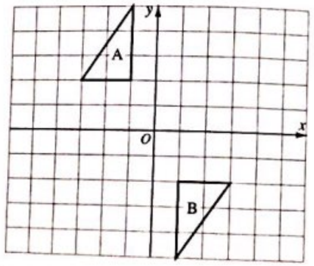
D.



E.



4. Which of the following transformations does NOT move triangle A on top of triangle B?



F. 180° rotation about the origin

G. Flip across the x-axis followed by a flip across the y-axis

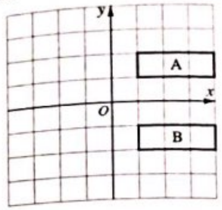
H. Flip across the y-axis followed by a flip across the x-axis

J. Slide 2 units right, followed by a flip across the line $x = 2$, followed by a flip across the x-axis

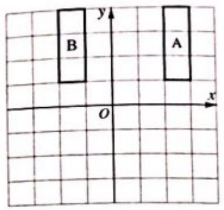
K. Slide 2 units left, followed by a flip across the line $x = -1$, followed by a flip across the x-axis

5. In which of the following choices can rectangle B NOT be created from rectangle A by reflections across the y-axis and/or reflections across the x-axis?

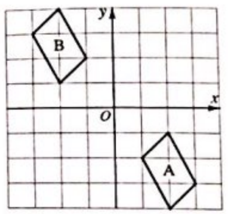
A.



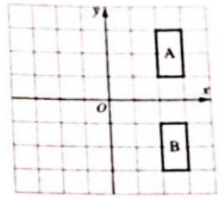
B.



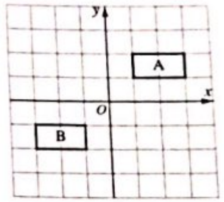
C.



D.



E.



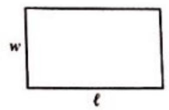
(Answers on page 388)

▪ Geometric Formulas

The ACT frequently gives formulas needed to answer a question. This section summarizes area and circumference formulas for reference.

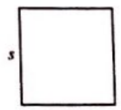
Rectangle

$A = \ell w$



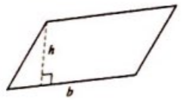
Square

$A = s^2$



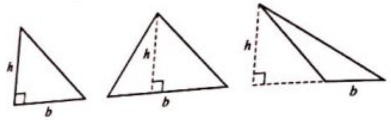
Parallelogram

$A = bh$



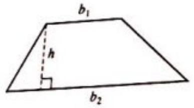
Triangle

$A = \frac{1}{2}bh$



Trapezoid

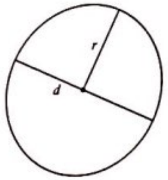
$A = \frac{1}{2}h(b_1 + b_2)$



Circle

$A = \pi r^2$

Circumference $= \pi d = 2\pi r$



The height must be perpendicular to the base. Look for the right angle symbol to identify the height.

CALCULATOR TIP

If an ACT question has answer choices in pi form (for example, 6π), check to see if your calculator can display answers in pi form. Otherwise, don't use a calculator.

MODEL ACT PROBLEM

What is the area of a circle that has a diameter equal to 6?

A. 36π
B. 18π
C. 12π
D. 9π
E. 6π

SOLUTION

Find the radius of the circle.

$r \text{ (radius)} = \frac{1}{2} \times d \text{ (diameter)}$

$r = \frac{1}{2} \times 6 = 3$

Find the area.

$A = \pi r^2$

$A = \pi(3)^2$

$A = \pi \times 9$

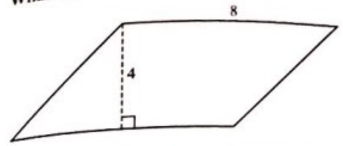
$A = 9\pi$

The correct answer is D.

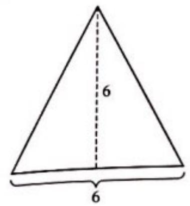
Practice

Where appropriate, use 3.14 for π.

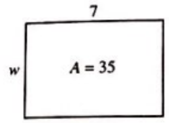
1. What is the area of the parallelogram below in square units?



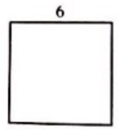
2. What is the height of a parallelogram if the base is 4 cm and the area is 20 cm²?
3. What is the area of this triangle in square units?



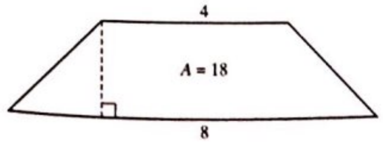
4. What is the base of a triangle if the area is equal to 24 in.² and the height is 6 in.?
5. What is the width of the rectangle?



6. What is the area of a rectangle if the length is 9 units and the width is two units less than the length?
7. What is the area of the square in square units?

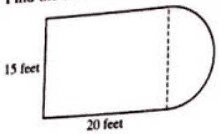


8. What is the length of a side of a square if the area is 121 square units?
9. What are the area and the circumference of a circle with radius equal to 5 cm?
10. What is the radius of a circle if the area is equal to 16π m²?
11. What is the height of this trapezoid?



12. What is the area of a trapezoid with height of 4 inches, one base equal to 6 inches, and the other base equal to twice the length of b_1?

13. Find the area of the rectangular deck with a semicircular end shown below.

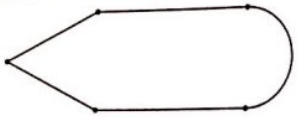


14. If the height of a triangle is 3.2 meters and the area is 6.4 square meters, what is the length of the base?

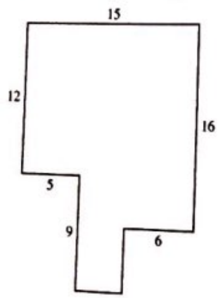
15. If the bases of a trapezoid measure 1.4 and 3.4 meters and the area is 19.2 square meters, what is the height of the trapezoid?

16. What is the radius of a circle with an area of 36π square units?

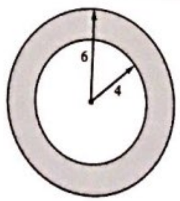
17. What is the area of the figure below, in pi form? It is a rectangle with a triangle at one end and a semicircle at the other end. The length of the rectangle is 10. The radius of the circle is 3. The height of the triangle is equal to its base.



18. What is the area of the figure below?



19. What is the area of the shaded region in the figure below in pi form?

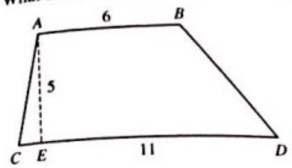


20. The side of a square is 8 units long. The square is cut and pieced together to form a rectangle with a width of 4 units. What is the length of this rectangle?

(Answers on page 388)

ACT-TYPE PROBLEMS

1. What is the area of the trapezoid shown below?



A. 22
B. 30
C. 34
D. 42.5
E. 66

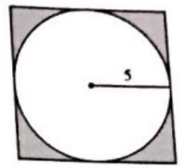
2. What is the circumference of a circle that has an area equal to 49π?

F. 7π
G. 14π
H. 21π
J. 28π
K. 49π

3. What is the area of a square whose diagonal has a length of 4 cm?

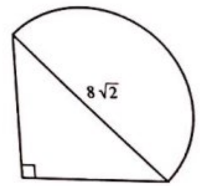
A. 2 cm²
B. $2\sqrt{2}$ cm²
C. 4 cm²
D. 8 cm²
E. 16 cm²

4. In the figure below, what is the area of the shaded region?



F. $25 - 5\pi$
G. $100 - 25\pi$
H. 10π
J. $100 - 10\pi$
K. 25π

5. The figure below is made up of a 45–45–90 triangle and a semicircle. Find the area of the entire figure.



A. $8 + 64\pi$
B. $8 + 128\pi$
C. $32 + 16\pi$
D. $32 + 32\pi$
E. $32 + 64\pi$

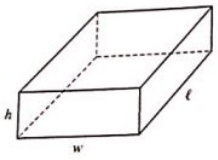
(Answers on page 388)

Geometry in Three Dimensions

Space occupies three dimensions and extends infinitely in all directions. Volume is a measure of how much space a three-dimensional figure takes up. The surface area of a three-dimensional figure is the total area occupied by the surface of the figure. Volume is expressed in cubic units. Surface area is expressed in square units.

Given on the next page are some three-dimensional figures with formulas for surface area and volume.

Rectangular Prism



A rectangular prism has 6 rectangular faces: two faces with dimensions ℓh, two faces with dimensions ℓw, and two faces with dimensions hw.

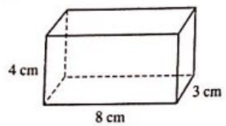
The surface area of a rectangular prism is the sum of the areas of the sides.

$SA = 2(\ell h + \ell w + hw)$

The volume of a rectangular prism is the product of the length, width, and height.

$V = \ell wh$

EXAMPLE



Find the surface area and volume of the rectangular prism above.

Write the dimensions.

$\ell = 8$ cm, $h = 4$ cm, $w = 3$ cm

Use the formulas.

Surface area

$SA = 2(\ell h + \ell w + hw)$

$SA = 2[(8 \text{ cm} \bullet 4 \text{ cm}) + (8 \text{ cm} \bullet 3 \text{ cm}) + (4 \text{ cm} \bullet 3 \text{ cm})]$

$SA = 2(32 \text{ cm}^2 + 24 \text{ cm}^2 + 12 \text{ cm}^2) = 136 \text{ cm}^2$

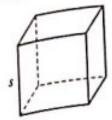
Volume

$V = \ell wh$

$8 \text{ cm} \bullet 3 \text{ cm} \bullet 4 \text{ cm} = 96 \text{ cm}^3$

Remember: Volume is measured in units^3.

Cube



A cube is a special rectangular prism whose faces are all identical squares. A cube has six faces. The area of each face is s^2.

The surface area of the cube is 6 times the area of a face.

$SA = 6s^2$

The length, width, and height are all equal, so the volume of the cube is the cube of one side.

$V = s^3$

EXAMPLE

What are the surface area and the volume of a cube with side length 5 cm?

$SA = 6s^2$

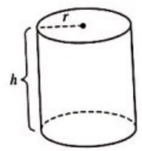
$SA = 6 \bullet 25 = 150 \text{ cm}^2$

The volume of a cube is the cube of one side.

$V = s^3$

$V = 5^3 = 125 \text{ cm}^3$

Cylinder



A cylinder is formed by two congruent, parallel, circular bases with radius r connected by a rectangular face with a height h.

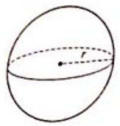
The surface area is the sum of the areas of the two circular bases, plus the area of the rectangular side. The length of the rectangle is equal to the circumference of one of the bases; the height is h.

$SA = 2\pi r^2 + 2\pi rh$

The volume of the cylinder is the area of the base multiplied by the height.

$V = \pi r^2 h$

Sphere



A sphere is all points a given distance (the radius, r) from a center point.

$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

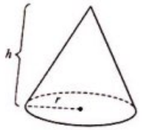
EXAMPLE

To the nearest tenth, what are the surface area and volume of a sphere with a radius of 4 cm? Use 3.14 for π.

$SA \approx 4 \cdot 3.14 \cdot 4^2 \approx 201.0 \text{ cm}^2$

$V \approx \frac{4}{3} \cdot 3.14 \cdot 4^3 \approx 267.9 \text{ cm}^3$

Cone



A cone consists of a circular base with a radius (r) and a single vertex a fixed height from the center of the base.

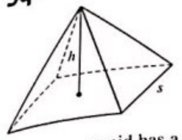
$V = \frac{1}{3}\pi r^2 h$

EXAMPLE

To the nearest tenth, what is the volume of a cone with a radius of 3 cm and a height of 6 cm? Use 3.14 for π.

$V \approx \frac{1}{3} \cdot 3.14 \cdot 3^2 \cdot 6 = 56.5 \text{ cm}^3$, to the nearest tenth.

Square Pyramid



A square pyramid has a square base and 4 congruent triangles as sides.

SA = the sum of the areas of the faces of the pyramid. Note that h above does not provide the heights of the triangles. Those measurements must be taken along the outside of the pyramid.

$V = \frac{1}{3}Bh$, where B is the area of the square base and h is the height from the base to the vertex.

MODEL ACT PROBLEMS

1. If the surface area of a cube is 96 cm², what is the length of each side of the cube?

   A. 3 cm
   B. 4 cm
   C. 5 cm
   D. 6 cm
   E. 7 cm

SOLUTION

Each face of a cube has the same area. Divide 96 cm² by 6, the number of faces, to find the area of one face of the cube. ($96 \text{ cm}^2 \div 6 = 16 \text{ cm}^2$)

Take the square root of the area of the face to find the length of the side.

$\sqrt{16 \text{ cm}^2} = 4$ cm. The length of each side of the cube is 4 cm.

The correct answer is B.

2. What is the radius of a sphere with a volume of $\frac{500\pi}{3}$?

   F. 125π
   G. 125
   H. 25
   J. 5π
   K. 5

Formula: $V = \frac{4}{3}\pi r^3$

Solve for r. $r^3 = \frac{V}{\frac{4}{3}\pi}$

$$r^3 = \frac{\frac{500\pi}{3}}{\frac{4\pi}{3}} = \frac{500\pi}{4\pi} = \frac{500}{4} = 125$$

$r = \sqrt[3]{125}$

$r = 5$

The correct answer is K.

Practice

Use 3.14 for π, and round your answer to the nearest tenth.

1. What is the surface area of a sphere with a radius of 5 cm?
2. What is the length of each side of a cube if the volume is 729 cm³?
3. What is the length of a rectangular prism with a volume of 576 cm³, a height of 6 cm, and a width of 8 cm?

4. What is the volume of a right circular cone if the height is 15 cm and the radius is 3 cm?

5. What is the radius of a right circular cylinder with a height of 5 cm and a volume of 62.8 cm³?

6. What is the volume of a cube that has a side equal to 4 units?

7. What is the length of a side of a cube whose volume is 216 cubic units?

8. What is the height of a rectangular prism whose volume is 350 cm³, length is 10 cm, and width is 7 cm?

9. What is the volume of a rectangular prism whose length is 4 units, width is 7 units, and height is 2 units?

10. What is the volume of a sphere that has a radius equal to 3 units?

11. What is the radius of a sphere that has a volume of 2,304π cubic centimeters?

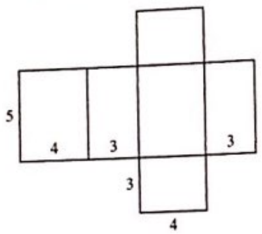
12. What is the volume of a cylinder in terms of π if the radius is 3 in. and the height is 9 in.?

13. What is the height of a cylinder that has a volume of 768π cubic units and a radius equal to 8 units?

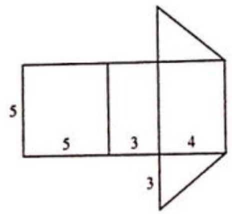
14. If a cylinder has a volume of 251.2 cm³ and a height of 5 cm, what is its diameter?

15. What is the volume of each storage box after assembly?

a.

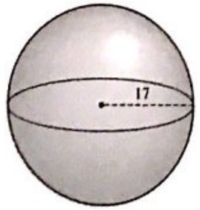


b.



16. A cylindrical can has a radius of 5 and a height of 10. What is the surface area of the can?

17. What is the volume of the sphere shown below?



18. A sphere has a radius of 12 units. A cylinder has a height of 9 units and the same volume as the sphere. What is the radius of the cylinder?

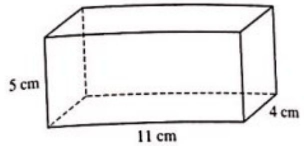
19. What is the volume of a cube if the length of a side is 10 inches?

20. The volume of a cube is 216 cm³. What is the surface area of the cube?

(Answers on page 389)

ACT-TYPE PROBLEMS

1. What is the surface area of the rectangular prism below?



A. 119 cm²
B. 220 cm²
C. 238 cm²
D. 357 cm²
E. 440 cm²

2. What is the area of each triangle in a square pyramid if the length of each side of the square base is 6 cm, and the total surface area is 132 cm²?

F. 20 cm²
G. 21 cm²
H. 22 cm²
J. 23 cm²
K. 24 cm²

3. If the volume of a sphere is 3,052.08 cm³, what is the radius of the sphere? (Use 3.14 for π.)

A. 7 cm
B. 8 cm
C. 9 cm
D. 10 cm
E. 11 cm

4. What is the volume of a cylinder with a height of 16 and a base area of 25π?

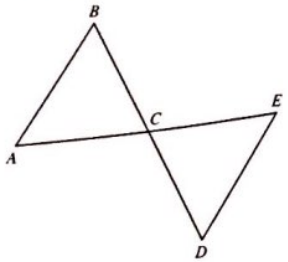
F. 5π
G. 25π
H. 50π
J. 200π
K. 400π

5. What is the area of a circle if it has the same radius as a sphere whose volume is 7,776π?

A. 18π
B. 36π
C. 81π
D. 324π
E. 1,296π

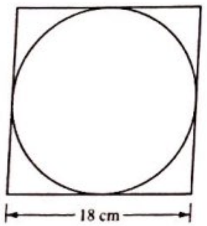
(Answers on page 389)

11. In the figure below, $\overline{BD}$ and $\overline{AE}$ bisect one another. Which of the following choices properly identifies two congruent triangles?



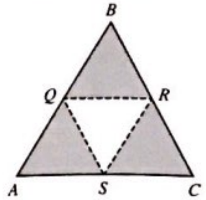
A. $\triangle ACB \cong \triangle ECD$
B. $\triangle ABC \cong \triangle DCE$
C. $\triangle ACB \cong \triangle DCE$
D. $\triangle BCA \cong \triangle ECD$
E. $\triangle CAB \cong \triangle CDE$

12. What is the volume of a sphere that has the same radius as the circle inscribed in the square below?



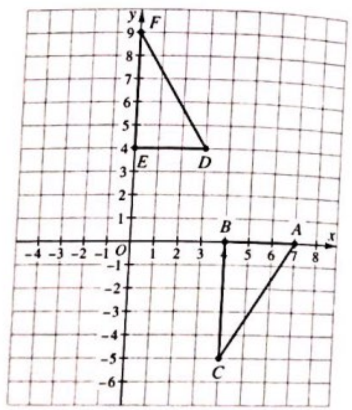
F. $8{,}748\pi$ cm³
G. $7{,}776\pi$ cm³
H. 972π cm³
J. $7{,}776\pi$ cm²
K. 972π cm²

13. $\triangle ABC$ is an equilateral triangle, each side having a length of 8 in. $\triangle QRS$ is formed by joining together the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. What is the area of the shaded region?



A. $2\sqrt{3}$ in.²
B. $4\sqrt{3}$ in.²
C. $6\sqrt{3}$ in.²
D. $8\sqrt{3}$ in.²
E. $12\sqrt{3}$ in.²

14. Which of the following transformations would place $\triangle ABC$ on top of $\triangle DEF$?



F. Horizontal slide −4 units, reflection across the line $y = 2$
G. Reflection across the line $x = 2$, vertical slide 8 units
H. Reflection across the line $y = 2$, horizontal slide −3 units
J. Reflection across the line $y = x$
K. Vertical slide 4 units up, rotation 180° about the point (4,4)

Chapter 12

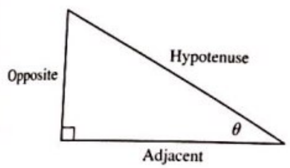
Trigonometry

- There are only four trigonometry problems on the entire ACT.
- Typically the ACT questions contain only right triangle or graph identification problems.
- The questions do not require an extensive knowledge of trigonometry.
- This review covers the material you will need to answer the trigonometry questions on the ACT.

Right Triangle Trigonometry

Right triangle trigonometry deals with angles less than 90°. The trigonometric functions are ratios of the sides of right triangles.

Here is the familiar right triangle with the names of the legs labeled.



The "opposite" side is opposite the angle θ. The "adjacent" side is adjacent to angle θ.

Trigonometric Functions

The basic trigonometric functions are given below. You should memorize these functions.

$$\text{sine of } \theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosine of } \theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{tangent of } \theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

You can use SOH-CAH-TOA to memorize these ratios. SOH-CAH-TOA: Sin is Opposite over Hypotenuse; Cos is Adjacent over Hypotenuse; Tan is Opposite over Adjacent.

CALCULATOR TIP

Scientific and graphing calculators allow you to compute trigonometric functions directly. However, the vast majority of the ACT trigonometry questions ask about your understanding of trigonometric relationships. The calculator will not help you answer these questions.

Inverse Trigonometric Functions

These three trigonometric functions are inverses of the previously mentioned three functions. You should memorize them.

cosecant of $\theta = \dfrac{1}{\text{sine of } \theta}$ $\qquad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

secant of $\theta = \dfrac{1}{\text{cosine of } \theta}$ $\qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

cotangent of $\theta = \dfrac{1}{\text{tangent of } \theta}$ $\qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

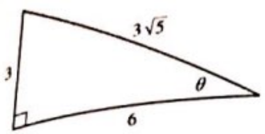
Recall that $\dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$

Angles can be measured using degrees or radians. The relationship between degrees (D) and radians (R) is $R = \dfrac{\pi}{180} \times D$.

It is helpful to memorize these common values of trigonometric functions.

θ	*Degrees* $0°$	*Radians* 0	*Degrees* $30°$	*Radians* $\frac{\pi}{6}$	*Degrees* $45°$	*Radians* $\frac{\pi}{4}$	*Degrees* $60°$	*Radians* $\frac{\pi}{3}$	*Degrees* $90°$	*Radians* $\frac{\pi}{2}$
$\sin\theta$	0		$\frac{1}{2}$		$\frac{\sqrt{2}}{2}$		$\frac{\sqrt{3}}{2}$		1	
$\cos\theta$	1		$\frac{\sqrt{3}}{2}$		$\frac{\sqrt{2}}{2}$		$\frac{1}{2}$		0	
$\tan\theta$	0		$\frac{\sqrt{3}}{3}$		1		$\sqrt{3}$		undefined	

EXAMPLES



1. Find the sin, cos, and tan of $\angle\theta$. Remember to simplify your answers.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{6} = \frac{1}{2}$$

2. Find the csc, sec, and cot of $\angle\theta$ in the triangle above.

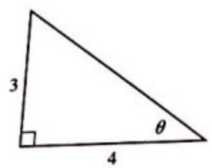
$$\sin\theta = \frac{\sqrt{5}}{5}, \text{ so } \csc\theta = \frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

$$\cos\theta = \frac{2\sqrt{5}}{5}, \text{ so } \sec\theta = \frac{5}{2\sqrt{5}} = \frac{5\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{2\cdot 5} = \frac{\sqrt{5}}{2}$$

$$\tan\theta = \frac{1}{2}, \text{ so } \cot\theta = 2$$

MODEL ACT PROBLEMS

1. In the triangle below, what is $\sin\theta$ = ?



A. $\frac{5}{3}$

B. $\frac{5}{4}$

C. $\frac{4}{5}$

D. $\frac{3}{4}$

E. $\frac{3}{5}$

SOLUTION

Find the length of the hypotenuse.

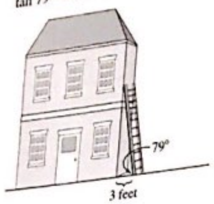
You may notice that this is a 3–4–5 triangle, with the hypotenuse equal to 5.

Find sin θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

The correct answer is E.

2. In the figure below, the base of the ladder is 3 feet from the house. If the ladder forms a 79° angle with the ground, what is the length of the ladder rounded to the nearest hundredth?
(*Note:* $\sin 79° \approx .981$, $\cos 79° \approx .191$, $\tan 79° \approx 5.145$)



F. 3.06 feet
G. 6.72 feet
H. 9.23 feet
J. 13.61 feet
K. 15.71 feet

SOLUTION

We know the length of the side adjacent to the angle and the measure of the angle. We want to find the length of the ladder, which is the hypotenuse.

$\theta = 79°$

adjacent = 3

hypotenuse = ?

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos 79° = \frac{3}{\text{hypotenuse}}$

Multiply both sides by the hypotenuse. $\quad \text{hyp} \times \cos 79° = 3$

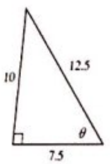
Divide both sides by cos 79°. $\quad \text{hyp} = \frac{3}{\cos 79°} \approx \frac{3}{.191}$

Use a calculator. $\quad \text{hyp} \approx 15.71$ feet

The correct answer is K.

Practice

In 1–6, find the trigonometric ratios for $\angle\theta$ in the triangle below.

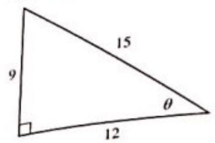


1. $\sin\theta =$
2. $\cos\theta =$
3. $\tan\theta =$
4. $\csc\theta =$
5. $\sec\theta =$
6. $\cot\theta =$
7. If $\tan\theta = \frac{5}{12}$, then $\sin\theta = ?$
8. If $\cot\theta = \frac{15}{8}$, then $\cos\theta = ?$
9. If $\sin\theta = \frac{12}{13}$, then $\csc\theta = ?$
10. If $\sec\theta = \frac{3}{2}$, then $\sin\theta = ?$

(*Answers on page 393*)

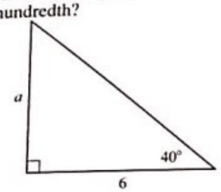
ACT-TYPE PROBLEMS

1. In the triangle below, $\sec\theta = ?$



A. $\frac{9}{12}$
B. $\frac{12}{15}$
C. $\frac{15}{12}$
D. $\frac{12}{9}$
E. $\frac{15}{9}$

2. In the triangle below, what is the length of side *a* rounded to the nearest hundredth?
(*Note:* $\sin 40° \approx .643$, $\cos 40° \approx .766$, $\tan 40° \approx .839$)

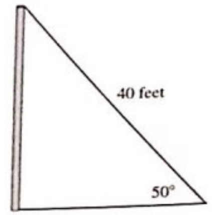


F. 5.38
G. 5.03
H. 4.60
J. 3.86
K. 3.05

3. If $\sin\theta = \frac{7}{25}$, then $\csc\theta = ?$

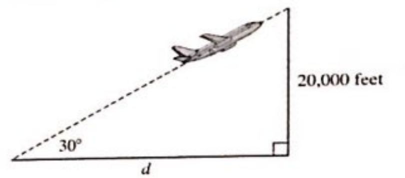
A. $\frac{7}{24}$
B. $\frac{24}{25}$
C. $\frac{25}{24}$
D. $\frac{24}{7}$
E. $\frac{25}{7}$

4. In the figure below, a rope is run from the top of the pole to the ground. The rope is 40 feet long and it forms a 50° angle with the ground. What is the height of the pole rounded to the nearest hundredth?
(*Note:* $\sin 50° \approx .766$, $\cos 50° \approx .643$, $\tan 50° \approx 1.192$)



F. 25.71 feet
G. 28.28 feet
H. 30.64 feet
J. 43.06 feet
K. 47.68 feet

5. An airplane takes off and climbs at a 30° angle to an altitude of 20,000 feet. To the nearest foot, what ground distance will the plane have flown when it reaches 20,000 feet?
(*Note:* $\sin 30° = .5$, $\cos 30° \approx .866$, $\tan 30° \approx .577$)



A. 17,321 feet
B. 20,000 feet
C. 23,094 feet
D. 34,662 feet
E. 40,000 feet

(*Answers on page 393*)

Trigonometric Identities

You can use these identities to find the values of other trigonometric ratios or to rewrite the ratios in equivalent forms. You should memorize them.

Reciprocal Identities

$\sec\theta = \frac{1}{\cos\theta}$ or $\sec\theta \cdot \cos\theta = 1$ or $\cos\theta = \frac{1}{\sec\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ or $\csc\theta \cdot \sin\theta = 1$ or $\sin\theta = \frac{1}{\csc\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ or $\cot\theta \cdot \tan\theta = 1$ or $\tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$

EXAMPLES

1. The sine of θ is $\sqrt{\frac{3}{4}}$. What is the cosine of θ?

   Use $\sin^2\theta + \cos^2\theta = 1$.

   Substitute. $\left(\sqrt{\frac{3}{4}}\right)^2 + \cos^2\theta = 1$

   Solve. $\frac{3}{4} + \cos^2\theta = 1$

   $\cos^2\theta = \frac{1}{4}$

   $\cos\theta = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

   The cosine of θ is $\pm\frac{1}{2}$.

2. The secant of θ is $\frac{5}{\sqrt{5}}$. What is the sine of θ?

   $\sec\theta = \frac{5}{\sqrt{5}}$, so $\cos\theta = \frac{\sqrt{5}}{5}$. (Secant and cosine are inverse functions.)

   Use $\sin^2\theta + \cos^2\theta = 1$.

   Substitute. $\sin^2\theta + \left(\frac{\sqrt{5}}{5}\right)^2 = 1$

   Solve. $\sin^2\theta + \frac{5}{25} = 1$

   $\sin^2\theta + \frac{1}{5} = 1$

   $\sin^2\theta = \frac{4}{5}$

   $\sin\theta = \sqrt{\frac{4}{5}}$

   $\sin\theta = \pm 2\sqrt{\frac{1}{5}}$

   The sine of θ is $\pm 2\sqrt{\frac{1}{5}}$.

3. Demonstrate that $2(1 - \sin^2\theta) - 1 = \cos 2\theta$.

   Use trigonometric identities to simplify the left side.

   $2(1 - \sin^2\theta) - 1$

   $= 2(\cos^2\theta) - 1$ Use the Pythagorean identity.

   $= 2\cos^2\theta - 1$ Remove parentheses.

   $= \cos 2\theta$ Use the double-angle identity.

MODEL ACT PROBLEMS

1. $\frac{\sin 2\theta}{\tan\theta} - 1 =$

   A. $\sin 2\theta$

   B. $\cos 2\theta$

   C. $\tan 2\theta$

   D. $\sin\frac{\theta}{2}$

   E. $\cos\frac{\theta}{2}$

SOLUTION

$$\frac{\sin 2\theta}{\tan\theta} - 1 = \frac{2\sin\theta\cos\theta}{\frac{\sin\theta}{\cos\theta}} - 1$$

$$= 2\sin\theta\cos\theta \times \frac{\cos\theta}{\sin\theta} - 1$$

$$= 2\cos^2\theta - 1 = \cos 2\theta$$

The correct answer is B.

2. Which of the following statements is FALSE?

   F. $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

   G. $\sec^2\theta - \tan^2\theta = 1$

   H. $(\csc^2\theta - \cot^2\theta)(\sin^2\theta + \cos^2\theta) = 1$

   J. $\tan 2\theta - \tan 2\theta\tan^2\theta = 2\tan\theta$

   K. $\cos 2\theta = 2\sin^2\theta - 1$

SOLUTION

The identity for $\cos 2\theta$ is: $\cos 2\theta = 1 - 2\sin^2\theta$.

Choice K, $\cos 2\theta = 2\sin^2\theta - 1$, cannot be derived from that identity. All of the other choices can be derived from an identity.

The correct answer is K.

Practice

1. If $\cos\theta = \frac{\sqrt{3}}{2}$, find the sine of θ using trigonometric identities.
2. If $\csc\theta = \sqrt{2}$, find the cosine of θ using trigonometric identities.
3. $\frac{1}{\tan\theta} \cdot \cot\theta + 1 = ?$
4. $\cos\theta\left(\frac{-\sin^2\theta}{\cos\theta} + \cos\theta\right) = ?$
5. $\pm\sqrt{\frac{1+\cos 2\theta}{2}} = ?$
6. $(2\cos^2\theta - 1)(1 - 2\sin^2\theta) = ?$
7. $\frac{\sin^2\theta}{\cos^2\theta} + 1 = ?$
8. $\pi(\sin^2\theta + \cos^2\theta) = ?$
9. $\frac{1-\cos\theta}{1+\cos\theta} = ?$
10. $\frac{\sec^2\theta}{1+\tan^2\theta} = ?$

(Answers on page 394)

ACT-TYPE PROBLEMS

1. $\frac{\frac{1}{1+\tan^2\theta}}{\sec\theta} \cdot 2\sin\theta = ?$

   A. $\csc^2\theta$

   B. $\tan 2\theta$

   C. $\sin 2\theta$

   D. $\sin\frac{\theta}{2}$

   E. $\sec^2\theta$

2. $\frac{\sin 2\theta}{2\cos\theta} \cdot \sin\theta + \cos^2\theta = ?$

   F. 1

   G. $\sec^2\theta$

   H. π

   J. $\tan 2\theta$

   K. $\tan^2\theta$

3. Which of the following statements is FALSE?

   A. $\sin^2\theta + \cos^2\theta = \csc^2\theta - \cot^2\theta$

   B. $\cos^2\theta - \sin^2\theta = \cos^2\theta + \sin^2\theta$

   C. $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

   D. $\frac{\tan^2\theta}{\sec^2\theta - 1} = 1$

   E. $\sec^2\theta - \csc^2\theta = \tan^2\theta - \cot^2\theta$

4. $1 - \frac{\sin^2\theta + \cos^2\theta}{\csc^2\theta} = ?$

   F. $\sin^2\theta$

   G. $\tan^2\theta$

   H. $\sec^2\theta$

   J. $\cos^2\theta$

   K. $\cot^2\theta$

5. $(2\cos^2\theta - 1) - (1 - 2\sin^2\theta) = ?$

   A. 0

   B. $\tan 2\theta$

   C. 1

   D. $\tan\frac{\theta}{2}$

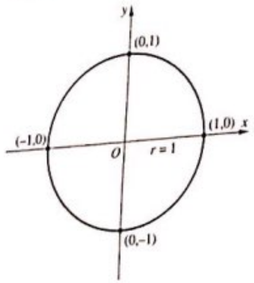
   E. $\tan^2\theta$

(Answers on page 394)

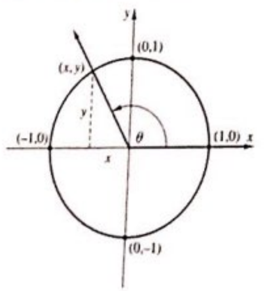
Unit-Circle Trigonometry

The right triangle is used as a reference for the trigonometric ratios of acute angles, angles less than 90°. A circle is used as a reference for trigonometric ratios of angles greater than or equal to 90°.

Draw a circle on the coordinate plane with the center at the origin and a radius of 1.



Draw an angle with one side on the x-axis. Start at point (1,0) and go around the circle counterclockwise to place the other side of the angle. You can use the coordinates on the unit circle to find trigonometric values.

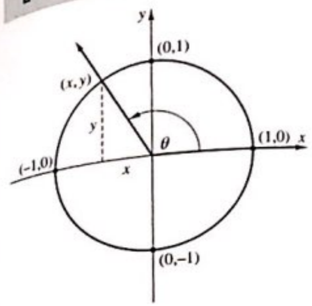


Notice that x and y will always be between -1 and $+1$. For the angle θ use the triangle with sides x and y to find the values of trigonometric ratios. The hypotenuse of this triangle always measures 1.

$\sin\theta = y$ $\qquad \cos\theta = x$ $\qquad \tan\theta = \frac{y}{x}\ (x \neq 0)$

$\csc\theta = \frac{1}{y}\ (y \neq 0)$ $\qquad \sec\theta = \frac{1}{x}\ (x \neq 0)$ $\qquad \cot\theta = \frac{x}{y}\ (y \neq 0)$

EXAMPLE



Find the sine, cosine, and tangent of angle θ in the unit circle above. Use the coordinates of the point $\left(x, \frac{1}{3}\right)$ on the circle that describes the angle.

We know that $\sin\theta = y$. In this example, $\sin\theta = \frac{1}{3}$. To find the cosine, use the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$.

$$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{9} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{8}{9}$$

$$\cos = \pm\sqrt{\frac{8}{9}}$$

$$\cos = \pm\sqrt{\frac{4 \times 2}{9}} = \pm\frac{2}{3}\sqrt{2}$$

$$x = -\frac{2}{3}\sqrt{2} \text{ (Take the negative root since } x \text{ is negative in the diagram.)}$$

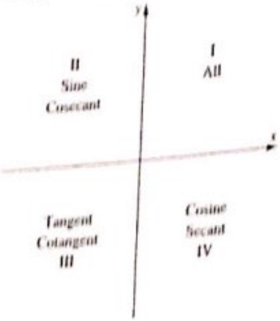
$$\tan\theta = \frac{y}{x}$$

$$= \frac{\frac{1}{3}}{-\frac{2}{3}\sqrt{2}}$$

$$= \frac{1}{-2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

Thus, $\sin\theta = \frac{1}{3}$, $\cos\theta = -\frac{2}{3}\sqrt{2}$, and $\tan\theta = -\frac{\sqrt{2}}{4}$.

When working with angles greater than 90° ($\theta > 90°$ or $\theta > \frac{\pi}{2}$) it is necessary to consider whether the trigonometric ratios are positive or negative. The coordinate grid below shows the quadrants in which the ratios are positive.

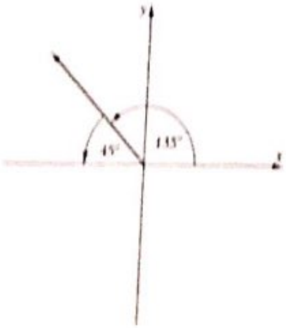


Quadrant I:	All ratios are positive.
Quadrant II:	Sine and cosecant are positive. All others are negative.
Quadrant III:	Tangent and cotangent are positive. All others are negative.
Quadrant IV:	Cosine and secant are positive. All others are negative.

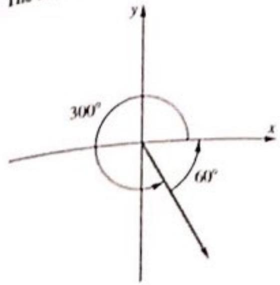
A reference angle is an angle in Quadrant I whose trigonometric ratios can be used to determine those of an angle in any other quadrant (angles greater than 90° or $\frac{\pi}{2}$). The reference angle can be found by calculating the difference between the given angle and 180° (π) or 360° (2π). Sometimes a problem may ask for a trigonometric function within a particular range of angle values. For example, $\frac{\pi}{2} < \theta < \pi$ means to find the function when θ falls in Quadrant II (between $\frac{\pi}{2}$ and π).

EXAMPLES

1. The reference angle for 135° is 180° − 135°, or 45°.



2. The reference angle for 300° is 360° − 300°, or 60°.



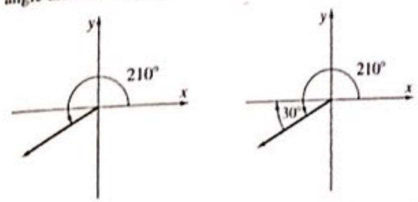
MODEL ACT PROBLEMS

1. What is the reference angle of a 210° angle?

A. 30°
B. 45°
C. 60°
D. 90°
E. 180°

SOLUTION

The number of degrees in the reference angle is the difference between the number of degrees in the given angle and the *x*-axis.



The 210° angle is 30° away from 180°. Therefore 30° is the reference angle for 210°.

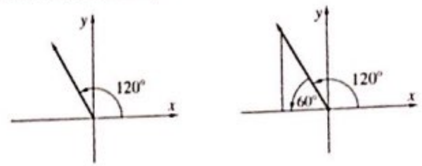
The correct answer is A.

2. cos 120° = ?

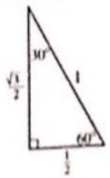
F. $-\frac{\sqrt{3}}{2}$
G. $-\frac{1}{2}$
H. $\frac{1}{2}$
J. $\frac{\sqrt{3}}{2}$
K. 2

SOLUTION

Find the reference angle.



120° is 60° away from 180°. Therefore the reference angle is 60°. Find cos 60°.



This is a 30-60-90 triangle so we know that

$$\cos 60° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2}$$

Find cos 120°.

Since 120° is in the second quadrant, the cosine of the angle is negative.

$$\cos 120° = -\frac{1}{2}$$

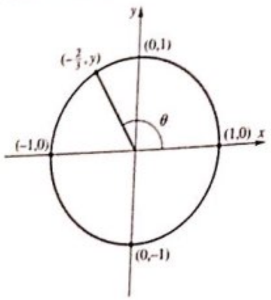
The correct answer is G.

Practice

What is the reference angle for each angle?

1. 125°
2. 315°
3. 425°
4. 156°

For 5–10, find the trigonometric ratios for angle θ in the circle below.



5. $\sin \theta =$
6. $\cos \theta =$
7. $\tan \theta =$
8. $\csc \theta =$
9. $\sec \theta =$
10. $\cot \theta =$

(Answers on page 394)

ACT-TYPE PROBLEMS

1. What is the reference angle of a 245° angle?
   - A. 25°
   - B. 35°
   - C. 45°
   - D. 55°
   - E. 65°

2. What is the reference angle of a 396° angle?
   - F. 26°
   - G. 36°
   - H. 46°
   - J. 56°
   - K. 66°

3. $\sin 300° = ?$
   - A. $\sqrt{3}$
   - B. $\frac{\sqrt{3}}{2}$
   - C. $\frac{1}{2}$
   - D. $-\frac{1}{2}$
   - E. $-\frac{\sqrt{3}}{2}$

4. $\cos 225° = ?$
   - F. $\frac{\sqrt{3}}{2}$
   - G. $\frac{1}{2}$
   - H. $\frac{\sqrt{2}}{3}$
   - J. $-\frac{\sqrt{2}}{2}$
   - K. $-\frac{\sqrt{3}}{2}$

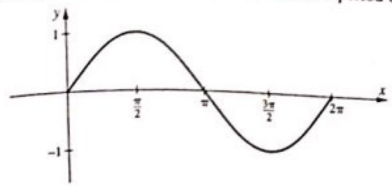
5. If $\sin \theta = -\frac{3}{5}$, and $\pi < \theta < \frac{3\pi}{2}$, then $\tan \theta = ?$
   - A. $-\frac{4}{5}$
   - B. $\frac{4}{5}$
   - C. $\frac{4}{3}$
   - D. $-\frac{3}{4}$
   - E. $\frac{3}{4}$

(Answers on page 394)

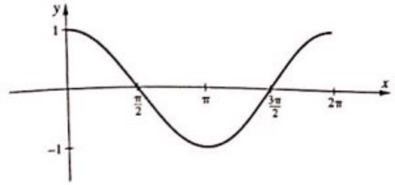
Graphs of Trigonometric Functions

Trigonometric functions repeat themselves. The **period** of a trigonometric function is the distance required to show one full cycle. You should be able to recognize the graphs of trigonometric functions. Those shown below are for one period of each function.

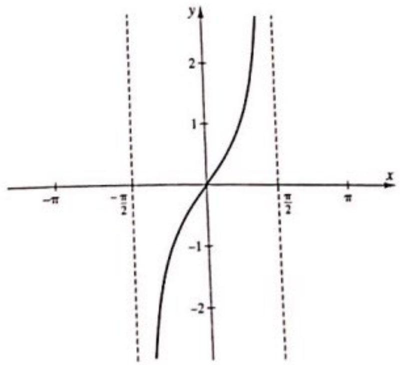
$y = \sin x$



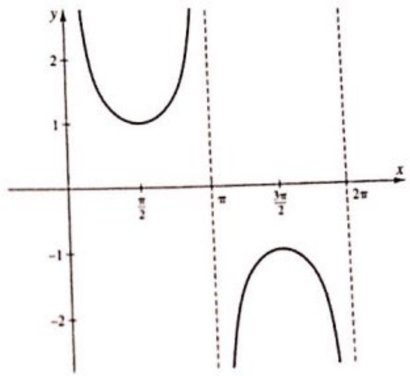
$y = \cos x$



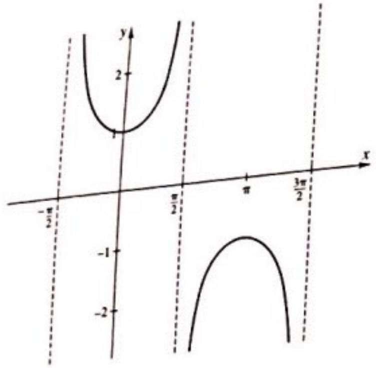
$y = \tan x$



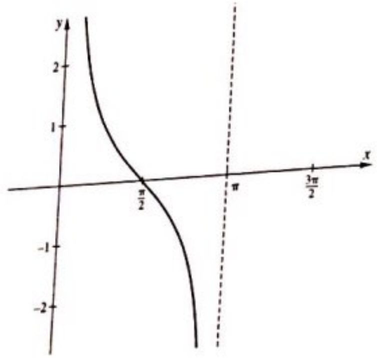
$y = \csc x$



$y = \sec x$



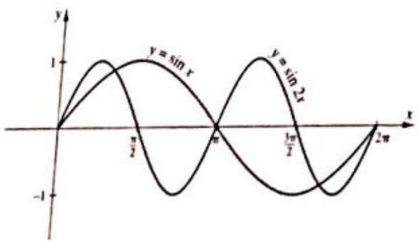
$y = \cot x$



The period of a trigonometric graph changes depending on the coefficient of x.

EXAMPLE

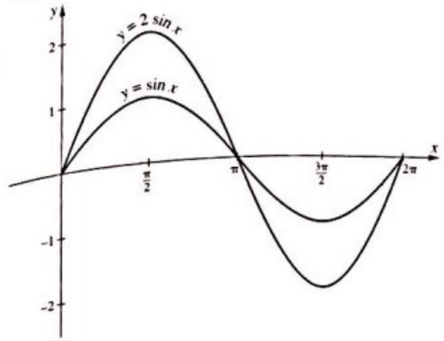
The function $y = \sin 2x$ has a period that is *half* as long as the period of $y = \sin x$.



The **amplitude** of a sine or cosine curve is half the distance between the smallest and largest y-values for the function. The amplitudes of the sine and cosine graphs change depending on the coefficient of sine or cosine. The amplitude for tangent, cotangent, secant, and cosecant is undefined.

EXAMPLE

The function $y = 2\sin x$ has an amplitude that is twice the amplitude of $y = \sin x$.



MODEL ACT PROBLEMS

1. What is the period of the graph $y = 4\tan 2x$?

   A. 4π
   B. 4
   C. π
   D. 2
   E. $\frac{\pi}{2}$

SOLUTION

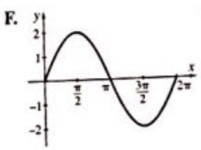
The period of $y = 4 \tan 2x$ is $\frac{\pi}{2}$.

Sketch the graph to convince yourself that the entire graph repeats after the interval $0 \le x \le \frac{\pi}{2}$.

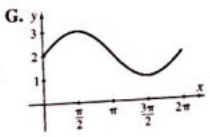
The correct answer is E.

2. Which graph below shows one period of $y = 2 \sin x$?

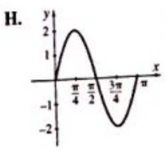
F.



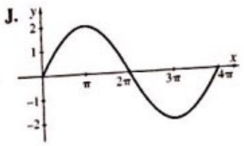
G.



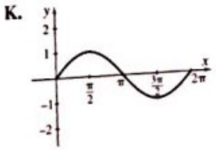
H.



J.



K.



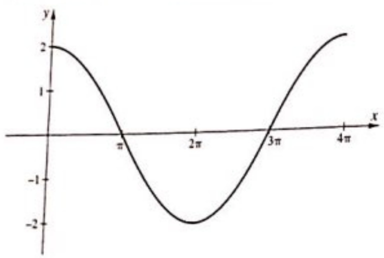
SOLUTION

The graph of $y = 2 \sin x$ looks just like the graph of $y = \sin x$, except that the amplitude is 2.

The correct answer is F.

Practice

Given below is one full period of the graph of a trigonometric function.

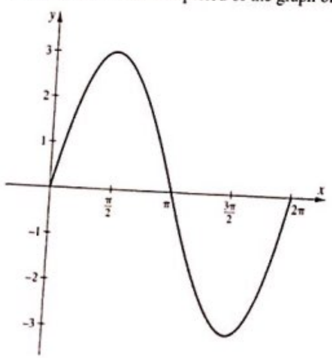


1. What is the amplitude of the graph?
2. What is the period of the graph?
3. What is the equation of the graph?

For 4–6, use the equation $y = \frac{1}{2}\csc x$.

4. What is the amplitude of the graph of this equation?
5. What is the period of the graph?
6. Draw a graph of one period of this function.

Given below is one full period of the graph of a trigonometric function.



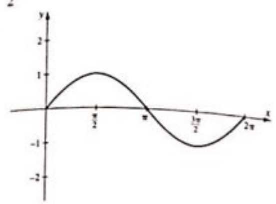
7. What is the amplitude of the graph?
8. What is the period of the graph?
9. What is the equation of the graph?
10. Draw a graph of one period of the function $y = -\cos 2x$.

(Answers on page 395)

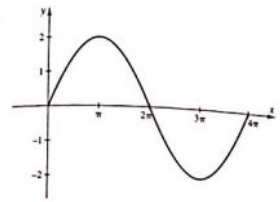
ACT-TYPE PROBLEMS

1. Which of the following is a graph of the function $y = \frac{1}{2}\sin 2x$?

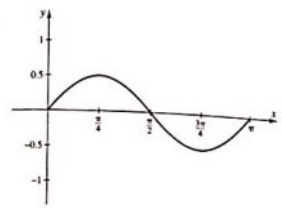
A.



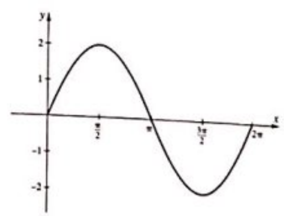
B.



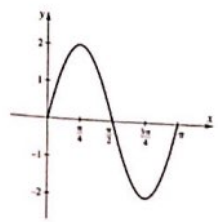
C.



D.



E.



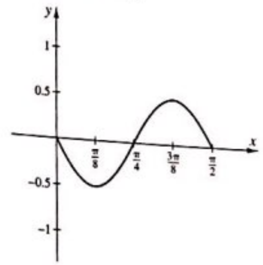
2. What is the product of the amplitude and period of the graph of the equation $y = 3\cos 2x$?

F. 15π
G. 6π
H. 3π
J. -3π
K. -6π

3. What is the period of the graph of the equation $y = -2\cot \pi x$?

A. $\frac{\pi}{2}$
B. 1
C. -1
D. $-\frac{\pi}{2}$
E. $-\pi$

4. Which of the following choices is the equation of the graph shown below?



F. $y = 2\sin 2x$
G. $y = -\frac{1}{2}\cos\frac{1}{2}x$
H. $y = -\frac{1}{2}\sin 4x$
J. $y = 4\sin\left(-\frac{1}{2}x\right)$
K. $y = 2\cos 2x$